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AGGREGATION ERRORS OF CELL-AVERAGED GEOID HEIGHT.(U)

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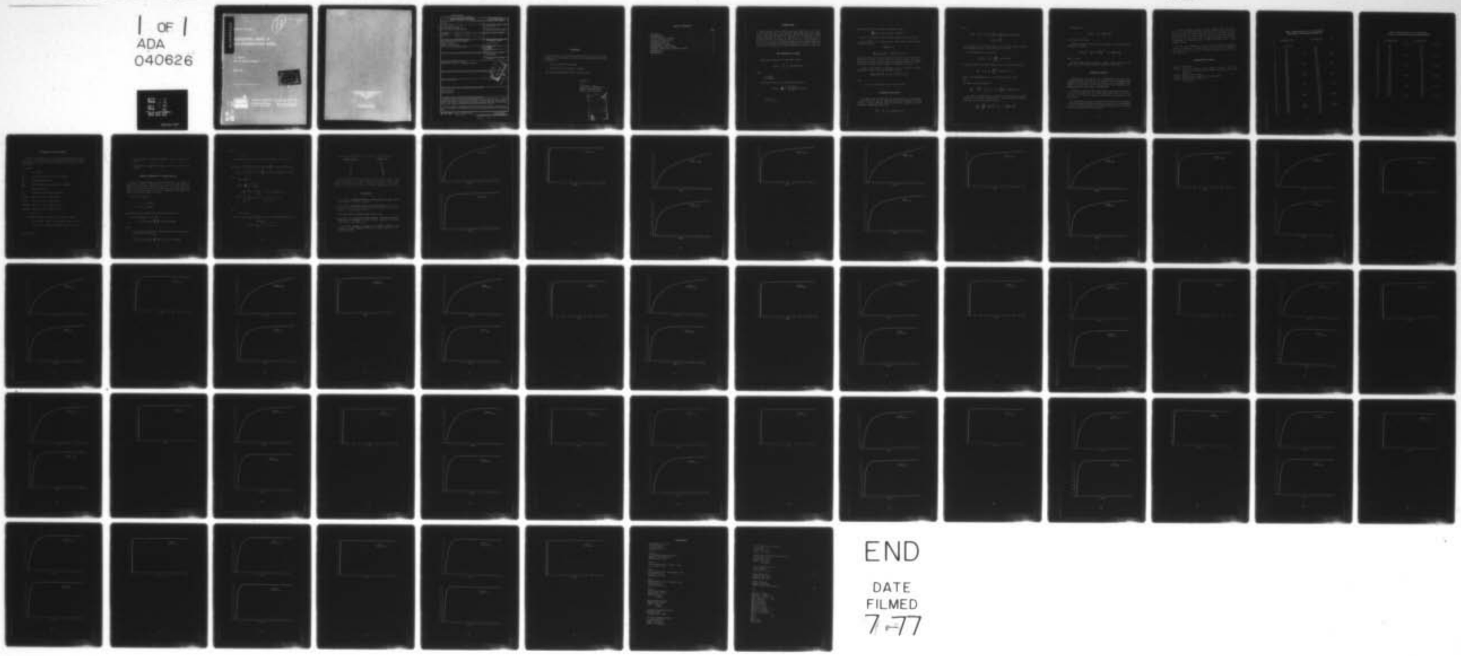

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AGGREGATION ERRORS OF CELL-AVERAGED GEOID HEIGHT

by

B. ZONDEK

Warfare Analysis Department

MAY 1977

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NAVAL SURFACE WEAPONS CENTER

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The earth's surface is divided into approximately equal cells bounded by lines of constant longitude and latitude. The geoid height is averaged over each of these cells. The mean degree variance of the error thus committed in the geoid height is calculated. Mean degree variances higher than of the 45th degree are found to be in error by over 100% for $5^\circ \times 5^\circ$ cells.

The error committed by calculating the cell-averaged geoid height from discrete altimetry ground tracks is also estimated. It is found to be less than 10 cm for more than 3 tracks for $5^\circ \times 5^\circ$ square sizes.

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FOREWORD

Satellite altimetry yields geoid height data along the ground tracks. These data are averaged over cells into which the earth's surface is divided. This report contains an investigation of:

1. The error committed by aggregation,

2. The error in the cell average due to sampling.

Dr. Edwin D. Ball has kindly agreed to review this report.

Released by:

RALPH A. NIEMANN, Head
Warfare Analysis Department

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INTRODUCTION

Satellite altimetry will soon yield extensive geoid height data over the oceans. One may divide the surface of the earth into approximately square cells of about equal area and average the geoid height within each cell. That would be a method of data aggregation. It leads to an imprecision in the determination of the anomalous field from geoid height. In order to make an a priori investigation of this effect, one may make the following hypothesis: The geoid height, without the J_2 term, is an homogeneous and isotropic random field over a spherical earth.[1,2] Its degree variances obey Kaula's rule.[3] Random quantities are underlined in this report.

THE MATHEMATICAL MODEL

The harmonic expansion of the geoid height is written

$$\underline{N}(\lambda,\phi) = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} \underline{Z}(m,\ell)U(m,\ell;\lambda,\phi)$$

where

λ is latitude
ϕ is longitude

The functions $U(m,\ell;\lambda,\phi)$ are surface harmonics defined by

$$U(m,\ell;\lambda,\phi) = \sqrt{(2\ell+1)\frac{(\ell-|m|)!}{(\ell+|m|)!}}\,P_\ell^{|m|}(\sin\lambda)e^{im\phi}$$

for

$\ell = 0, 1, \cdots$
$m = -\ell, -\ell+1, \cdots, \ell$

These harmonics satisfy the following orthonormality relations,

$$\iint U(m',\ell';\lambda,\phi)^* U(m,\ell;\lambda,\phi)d\Omega = 4\pi\delta(m,m';\ell,\ell')$$

They have been defined so that their mean square over the sphere is unity.

The coefficients in the expansion of the geoid height are random variables satisfying the relations,

$$E\left\{\underline{Z}(m,\ell)\right\} = 0$$

$$E\left\{\underline{Z}(m,\ell)\underline{Z}(m',\ell')^*\right\} = \frac{10^{-10}R^2}{\ell^4}\,\delta(m,m';\ell,\ell')$$

according to *Kaula's rule* (R is the nominal radius of the earth). A discussion of different degree variance attenuation laws can be found in Rapp.[4] The assumed homogeneity and isotropy of the random function $\underline{N}(\lambda,\phi)$ defined over the sphere, implies that the harmonic coefficients $\underline{Z}(m,\ell)$ are uncorrelated as indicated.

It may be noted, although we shall make no use of it, that the geoid height autocorrelation function based on our hypothesis is given by

$$E\left\{\underline{N}(\lambda,\phi)\underline{N}(\lambda',\phi')\right\} = 10^{-10}R^2 \sum_{\ell=2}^{\infty} \frac{2\ell+1}{\ell^4} P_\ell(\cos\psi)$$

where

ψ is the arc length between the rays (λ,ϕ) and (λ',ϕ').

AVERAGING OVER CELLS

The earth's surface is divided into zones bounded by lines of constant latitude at equal intervals. The zones are divided into equal cells along lines of constant longitude. A simple consideration shows that the harmonic expansion of the error geoid height corresponding to these cells is

$$\underline{\Delta}N = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \underline{L}(m,\ell)U(m,\ell;\lambda,\phi)$$

where

$$\underline{L}(m,\ell) = \sum_{\ell'=2}^{\infty} \sum_{m'=-\ell'}^{\ell'} \underline{Z}(m',\ell') \left\{ \sum_{I} \sum_{J} \frac{4\pi}{AA(I)} K(I,J;m',\ell')^* K(I,J;m,\ell) - \delta(m,m';\ell,\ell') \right\}$$

I and J denote the zones and cells within a zone, respectively. AA(I) is the solid angle subtended by a cell in zone I at the centre of the earth.

The quantities $K(I,J;m,\ell)$ are defined by

$$K(I,J;m,\ell) = \frac{1}{4\pi} \iint_{Cell(I,J)} U(m,\ell;\lambda,\phi)^* d\Omega$$

It follows that the error anomalous potential is given by (using Bruns' equation),

$$\underline{\Delta T} = \gamma \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left(\frac{R}{r}\right)^{\ell+1} \underline{L}(m,\ell) U(m,\ell;\lambda,\phi)$$

where r is the radial distance and γ is the acceleration of gravity on the earth.

The radial error gravity disturbance is

$$\underline{\delta\Delta g_r} = -\frac{\partial \underline{\Delta T}}{\partial r} = \frac{\gamma}{R} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} (\ell+1) \left(\frac{R}{r}\right)^{\ell+2} \underline{L}(m,\ell) U(m,\ell;\lambda,\phi)$$

The random field $\underline{\Delta N}$ defined on the sphere is not homogeneous and isotropic. Nevertheless, in view of the orthogonality of the harmonic functions, one obtains a simple expression for its mean variance, namely

$$E\left\{\frac{1}{4\pi} \iint_{sphere} \underline{\Delta N}^2 d\Omega\right\} = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} E\left\{|\underline{L}(m,\ell)|^2\right\}$$

The expressions

$$\sigma^2(\ell;\Delta N) = \sum_{m=-\ell}^{\ell} E\left\{|\underline{L}(m,\ell)|^2\right\}$$

are the mean degree variances.

Similarly, one obtains for the mean degree variances of the radial error gravity disturbance, the expression

$$\sigma^2(\ell;H,\Delta\delta g_r) = \frac{\gamma^2}{R^2}(\ell+1)^2\left(\frac{R}{r}\right)^{2\ell+4} \sum_{m=-\ell}^{\ell} E\left\{|\underline{L}(m,\ell)|^2\right\}$$

where $r = R + H$.

The mean degree variances of $\underline{\Delta N}$ are a measure – within our model – of the violence done to the anomalous field by the operation of cell averaging.

NUMERICAL RESULTS

Calculations have been made for a $5° \times 5°$ partitioning of the earth's surface considering harmonics up to degree 60. For comparison, the corresponding quantities pertaining to the Kaula field have also been calculated. A recursive algorithm for the integrals of the associated Legendre functions needed for the calculation of the quantities $K(I,J;m,\ell)$, was devised by A. R. DiDonato.[5]

In Table 1 we present the degree variances and accumulated degree variances of the error geoid height and the nominal Kaula geoid height. The factor γ (γ = 978 cm $\times$ s^{-2}) has been included to convert these into potential units. The units are cgs.

The graphs located on pages 12 through 44 of this report present the logarithm of the error potential and nominal potential at the surface of the earth cumulatively as a function of harmonic degree. The corresponding logarithm of the radial gravity disturbance at different altitudes is also plotted.

One observes that the low degree variances, of $\underline{\Delta N}$ are small. The reason is that the low degree harmonics are little affected by the $5° \times 5°$ cell averaging. For higher degrees they become larger and finally exceed the nominal Kaula values. The corresponding harmonics are represented with an error exceeding their magnitude. However, the sum of the degree variances of $\underline{\Delta N}$ is less than that of $\underline{N}$ ($\sim 40^2 m^2$). Asymptotically, the mean degree variances of $\underline{\Delta N}$ attenuate like const/n^2 – . Those of $\underline{N}$ like const/n^3.

As to the gravity disturbance, one observes that, for example, at an altitude of 500 km it is represented to a relative precision of about 1/20. At low altitudes the low degree mean variances are well represented, the high degree ones, poorly – as one would expect.

DESCRIPTION OF TABLE 1

Column 1	The degree ℓ
Column 2	Mean degree variance of error potential in units of cm^4/s^4. The harmonics are so normed that the integral of their square over the unit sphere is 4π.
Column 3	Progressive sum of column 1.
Column 4	Degree variance according to Kaula's attenuation law.
Column 5	Progressive sum of column 4.
	$\gamma = 978$ cm/s^2

Table 1. Degree Variance of 5° × 5° Cell-Averaged Potential and of the Nominal Kaula Potential

ℓ	$\gamma^2\sigma^2(\ell;\Delta N) \times 10^{-9}$	Σ	$\gamma^2\sigma^2(\ell;N) \times 10^{-9}$	Σ
0				
1	$.199 \times 10^{-6}$			
2	.175		9703	
3	.193		2875	
4	.217		1213	
5	.243	.828	621	14412
6	.270		359	
7	.296		226	
8	.323		152	
9	.349		106	
10	.374	2.439	77.6	15333
11	.398		58.3	
12	.422		44.9	
13	.445		35.3	
14	.467		28.3	
15	.487	4.658	23.0	15523
16	.507		19.0	
17	.526		15.8	
18	.544		13.3	
19	.561		11.3	
20	.576	7.372	9.70	15591.8
21	.591		8.38	
22	.605		7.29	
23	.618		6.38	
24	.630		5.62	
25	.642	10.457	4.97	15624.5
26	.652		4.42	
27	.662		3.94	
28	.671		3.54	
29	.681		3.18	
30	.689	13.813	2.87	15642.41
31	.698	14.512	2.61	
32	.706		2.37	
33	.715		2.16	
34	.723		1.97	

Table 1. Degree Variance of 5° × 5° Cell-Averaged Potential and of the Nominal Kaula Potential (Continued)

ℓ	$\gamma^2\sigma^2(\ell;\Delta N) \times 10^{-9}$	Σ	$\gamma^2\sigma^2(\ell;N) \times 10^{-9}$	Σ
35	.732	17.388	1.81	15653.33
36	.741		1.66	
37	.752		1.53	
38	.761		1.41	
39	.773		1.31	
40	.784	21.199	1.21	15660.47
41	.800		1.13	
42	.813		1.05	
43	.831		.976	
44	.848		.911	
45	.871	25.361	.852	15665.38
46	.891		.797	
47	.919		.748	
48	.945		.702	
49	.980		.660	
50	1.013	30.109	.621	15668.907
51	1.055		.585	
52	1.097		.552	
53	1.150		.521	
54	1.203		.493	
55	1.271	35.886	.467	15671.525
56	1.340		.442	
57	1.426		.419	
58	1.519		.398	
59	1.633		.378	
60	1.764	43.569	.359	15673.521

DESCRIPTION OF THE GRAPHS*

The units of all quantities are cgs, except that height above the earth is reckoned in km. The degree variances correspond to harmonics normed, so that their mean square over the sphere is unity. The nominal field is assumed to obey Kaula's attenuation law.

Notation:

$\underline{N}$	Geoid height
$\underline{\Delta N}$	Error in geoid height due to 5° X 5° aggregation
$\underline{\delta g_r}$	Radial gravity disturbance
$\underline{\Delta \delta g_r}$	Error in radial gravity disturbance due to aggregation
H	Height above Earth
L, ℓ	Maximal and running degrees of harmonics
$\sigma^2(\ell;\Delta N)$	Mean degree variance of $\underline{\Delta N}$ of degree ℓ
$\sigma^2(\ell;N)$	Mean degree variance of $\underline{N}$ of degree ℓ
$\sigma^2(\ell;H,\Delta\delta g_r)$	Mean degree variance of $\underline{\Delta\delta g_r}$ of degree ℓ
$\sigma^2(\ell;H,\delta g_r)$	Mean degree variance of $\underline{\delta g_r}$ of degree ℓ

$\gamma = 978 \text{ cm/s}^2$

The following logarithmic cumulative degree variances are plotted:

1. $\log \gamma^2 \sigma^2(L;\Delta N) = \log\left\{\gamma^2 \sum_{\ell=0}^{L} \sigma^2(\ell;\Delta N)\right\}$ as a function of degree L.
2. $\log \gamma^2 \sigma^2(L;N) = \log\left\{\gamma^2 \sum_{\ell=0}^{L} \sigma^2(\ell;N)\right\}$ as a function of degree L.

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*Graphs start on page 12.

3. $\log \sigma^2(L, H; \Delta\delta g_r) = \log \left\{ \sum_{\ell=0}^{L} \sigma^2(\ell; H, \Delta\delta g_r) \right\}$ as a function of degree L and height H.

4. $\log \sigma^2(L, H; \delta g_r) = \log \left\{ \sum_{\ell=0}^{L} \sigma^2(\ell; H, \delta g_r) \right\}$ as a function of degree L and height H.

SAMPLING ERRORS DUE TO TRACK SPACING

Altimetry yields geoid height data along the ground tracks of the vehicle. We may consider the following simplified problem. A cell on the earth, bounded by latitude and longitude lines, is covered by a number of evenly spaced ground tracks parallel to one of the boundaries. Along these tracks geoid height is given. We estimate the mean geoid height over the cell by averaging these data. What is the variance of the resulting estimate of the mean?

Let the cell be defined by

$$-\alpha \leqslant \phi \leqslant \alpha \quad \text{(longitude)}$$

$$-\alpha \leqslant \lambda \leqslant \alpha \quad \text{(latitude)}$$

and let there be k tracks parallel to the meridian and equally spaced.

The mean geoid height over the cell is

$$\underline{\tilde{N}} = \frac{1}{A} \sum_{\ell} \sum_{m} \underline{Z}(m,\ell) \int_{-\alpha}^{\alpha} \int_{-\alpha}^{\alpha} U(m,\ell;\lambda,\phi) \cos\lambda d\lambda d\phi$$

where

A is the solid angle subtended by the cell at the centre of the earth. The average over the track values is

$$\underline{\hat{\tilde{N}}} = \frac{1}{A} \sum_{\ell} \sum_{m} \underline{Z}(m,\ell) \sum_{r=1}^{k} \Delta \int_{-\alpha}^{\alpha} U(m,\ell;\lambda - \alpha + (r - \tfrac{1}{2})\Delta) \cos\lambda d\lambda$$

where

$$\Delta = 2\alpha/k$$

One obtains after a few steps the following expression for the error variance

$$E\left\{(\underline{\tilde{N}} - \underline{\hat{N}})^2\right\} = \frac{10^{-10}R^2}{A^2}\left\{\sum_{\ell=1}^{\infty}\frac{4}{(2\ell)^4}\sum_{s=0}^{\ell}\left[N(2s,2\ell)\int_0^{\alpha}P_{2\ell}^{2s}(\sin\lambda)\cos\lambda d\lambda\right]^2 f(\alpha,k,2s)^2\right.$$
$$\left. + \sum_{\ell=1}^{\infty}\frac{4}{(2\ell+1)^4}\sum_{s=0}^{\ell}\left[N(2s+1,2\ell+1)\int_0^{\alpha}P_{2\ell+1}^{2s+1}(\sin\lambda)\cos\lambda d\lambda\right]^2 f(\alpha,k,2s+1)^2\right\}$$

where

$$N(0,\ell) = \sqrt{2\ell+1}$$

$$N(\dot{m},\ell) = \sqrt{2(2\ell+1)\frac{(\ell-m)!}{(\ell+m)!}}$$

$$f(\alpha,k,m) = \begin{cases} \dfrac{2\sin m\alpha}{m} - \dfrac{2\alpha}{k}\dfrac{\sin m\alpha}{\sin\dfrac{m\alpha}{k}} & \text{for } \dfrac{m\alpha}{k} \text{ not a multiple of } \pi \\ \dfrac{2\sin m\alpha}{m} - 2\alpha(-1)^{r(1-k)} & \text{for } m \neq 0,\ \dfrac{m\alpha}{k} = r\pi \\ 0 & \text{for } m = 0 \end{cases}$$

where

r is any integer

One may easily establish that $E\left\{\underline{\tilde{N}}^2\right\}$ is obtained by replacing the function f by

$$g(\alpha,k,m) = \begin{cases} \dfrac{2\sin m\alpha}{m} & \text{for} \quad m \neq 0 \\ 2\alpha & \text{for} \quad m = 0 \end{cases}$$

The results calculating with a $5° \times 5°$ square are in the following table

Number of Tracks	$\sigma(\underset{\sim}{\tilde{N}} - \hat{\underline{N}})$ in cm
1	66
2	18
3	6.1
4	2.7
5	1.7
6	1.14
7	.84
10	.41
20	.10

The standard deviation of $\tilde{\underline{N}}$ comes out to be 38 m, which is almost the full Kaula value, because of the relative smallness of the cell. We see that a small number of well placed tracks leads to a negligible sampling error. The principal sources of error remain the uncertainty in the satellite ephemeris and the sea state.

REFERENCES

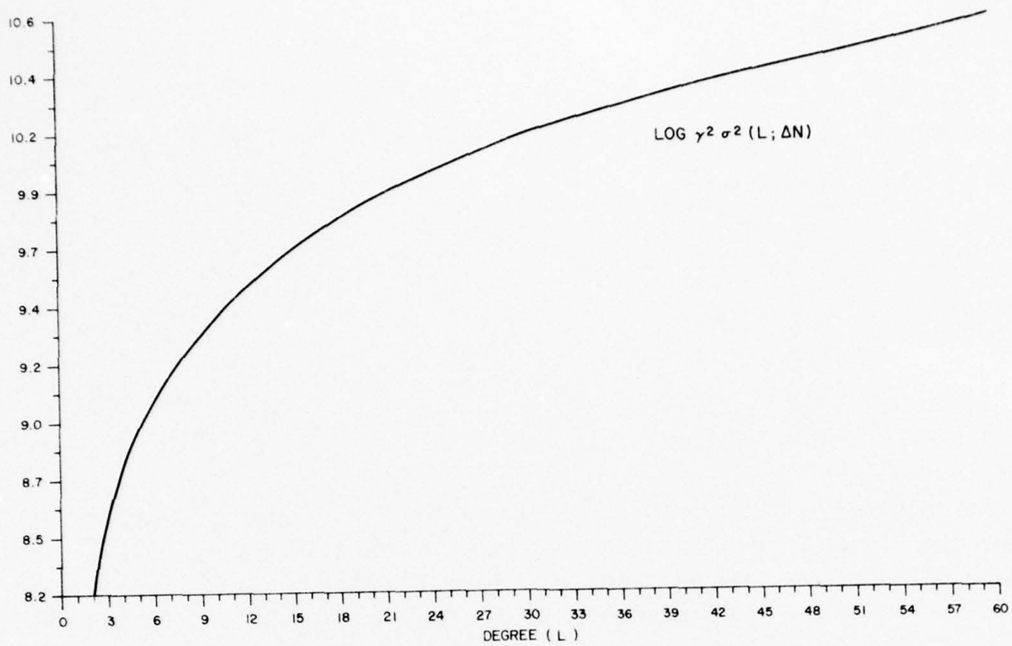
1. A. M. Obukhov, ***Statistically Homogeneous Random Fields on a Sphere***, Uspehi Mat. Nauk, **2**, No. 2 (1947), pp. 196-198.

2. A. M. Yaglom, ***Second-Order Homogeneous Random Fields***, Proc. of the 4th Berkeley Symposium on Mathematical Statistics and Probability, **2**, pp. 593-622, Berkeley, U. of California Press (1961).

3. W. M. Kaula, ***Theory of Satellite Geodesy***, Blaisdell (1966).

4. Richard H. Rapp, ***Geopotential Coefficient Behaviour to High Degree and Geoid Information by Wavelength***, Ohio State University, Department of Geodetic Science, Rpt. No. 180 (August 1972).

5. A. R. DiDonato, ***Recurrence Relations for the Indefinite Integrals of the Associated Legendre Functions***, NSWC/DL Technical Memo No. 5/77 (Publication Pending).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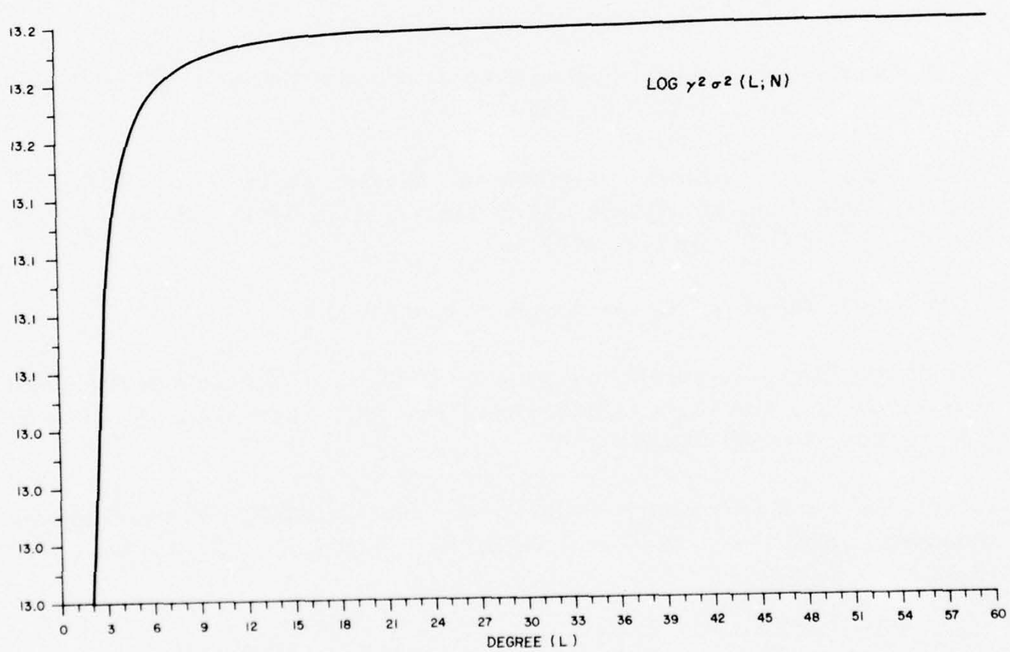
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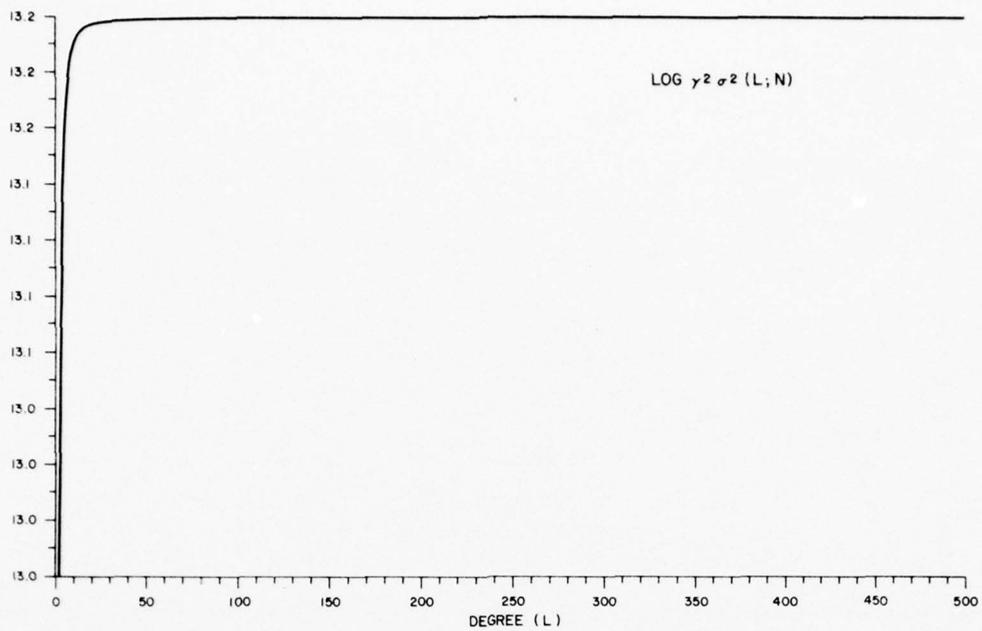

LOG $\gamma^2 \sigma^2$ (L; N)
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13.2
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13.1
13.1
13.1
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300
350
400
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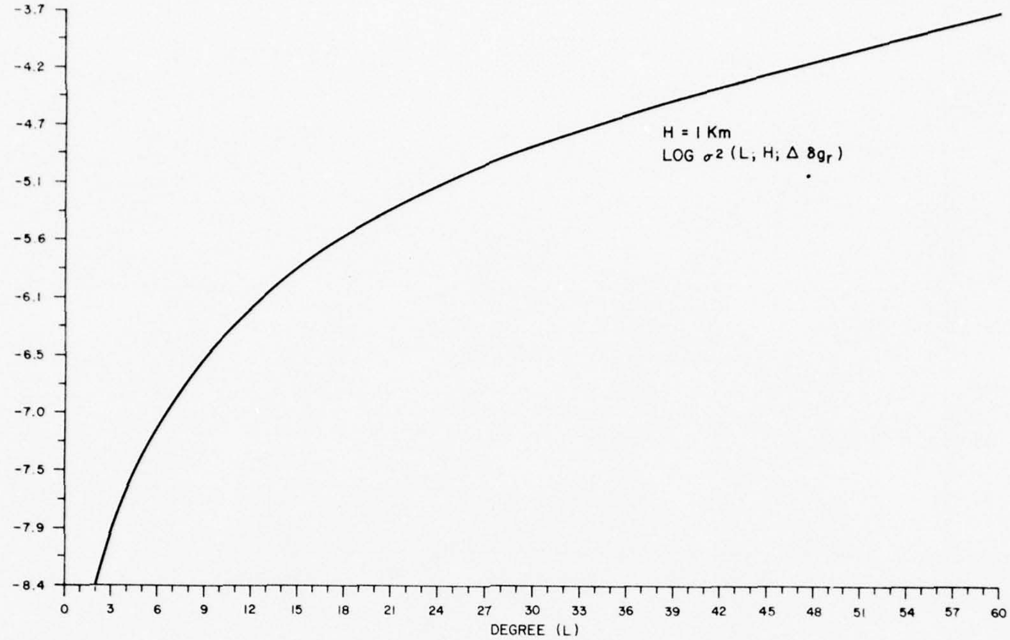
500
DEGREE (L)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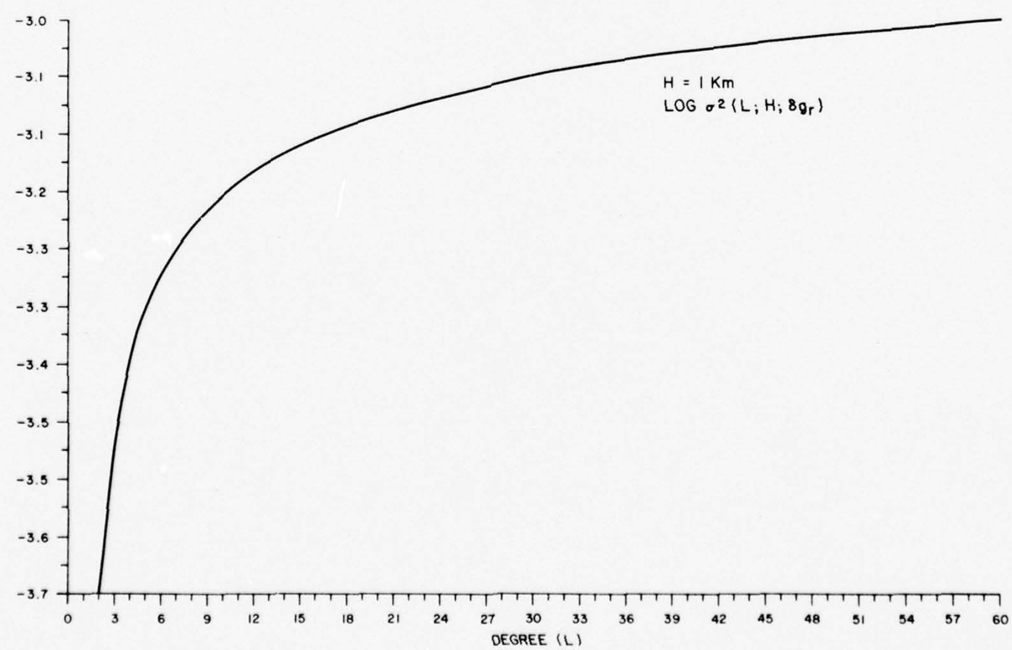
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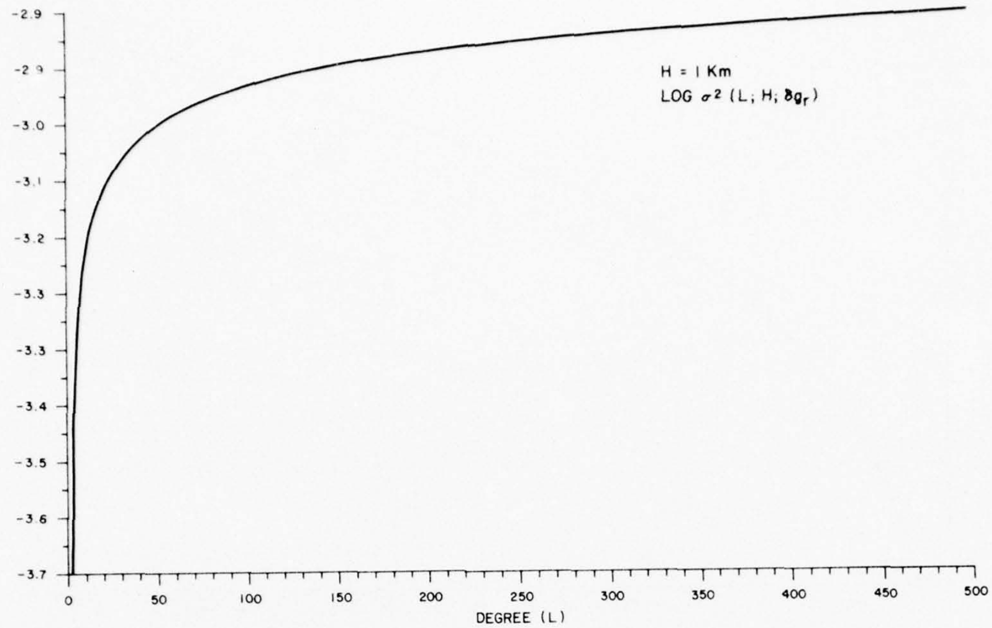

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-3.2
-3.3
-3.3
-3.4
-3.5
-3.6
-3.7
0
50
100
150
200
250
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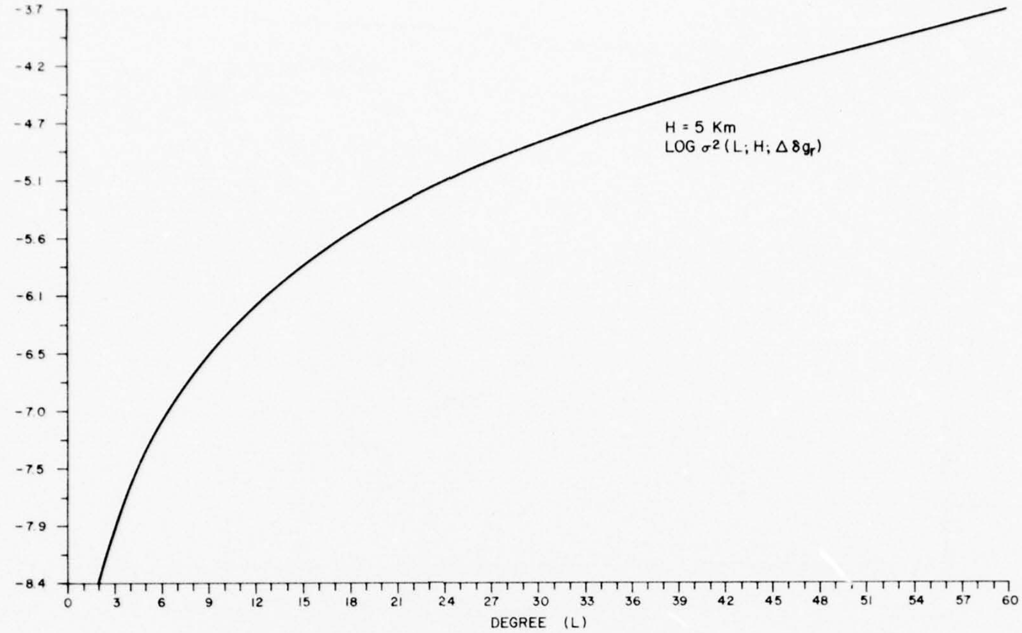
350
400
450
500
DEGREE (L)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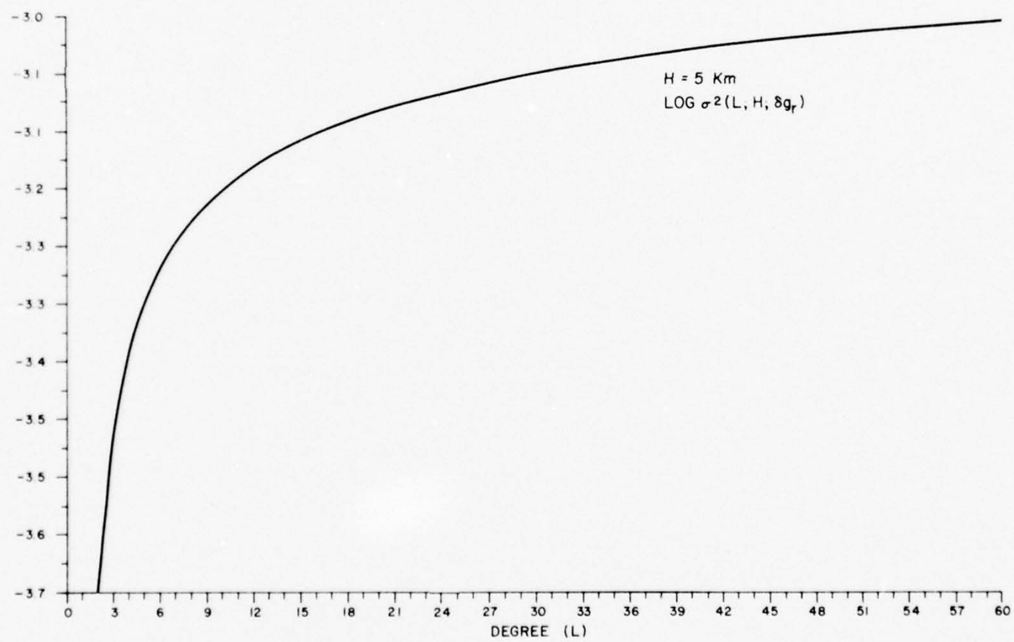
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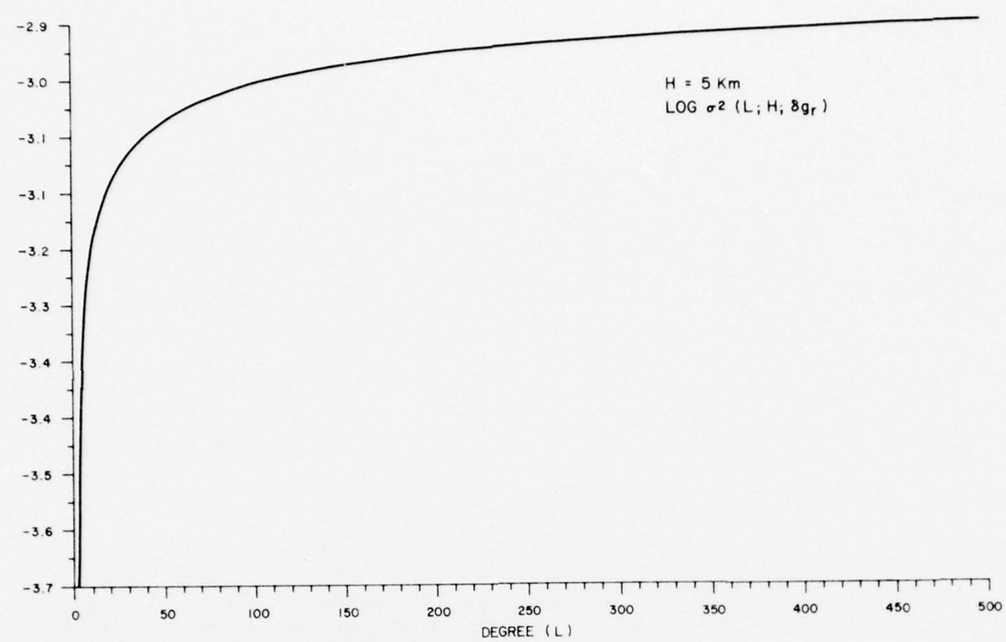
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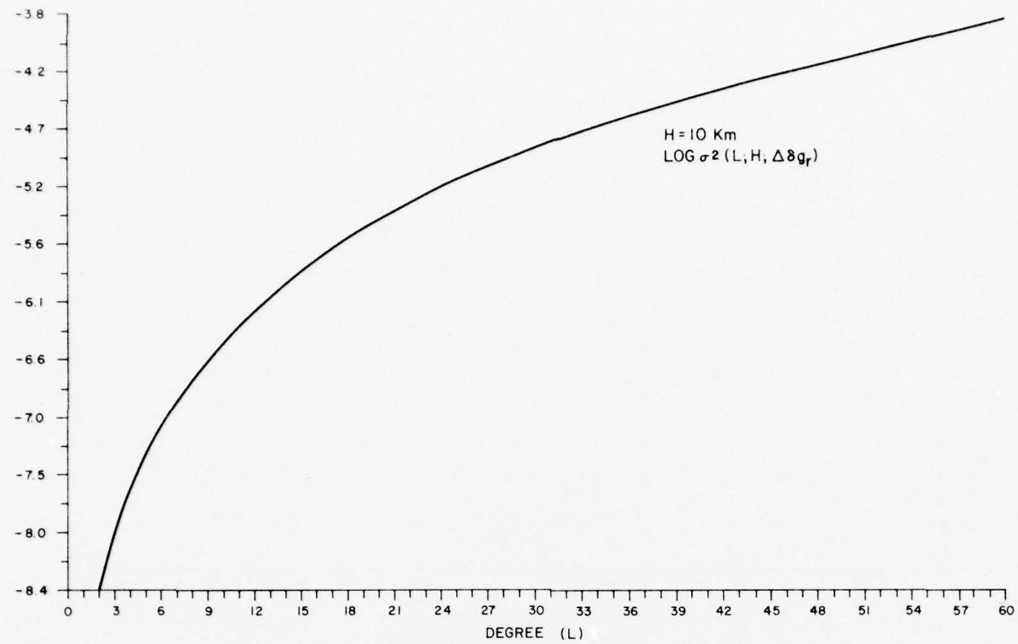

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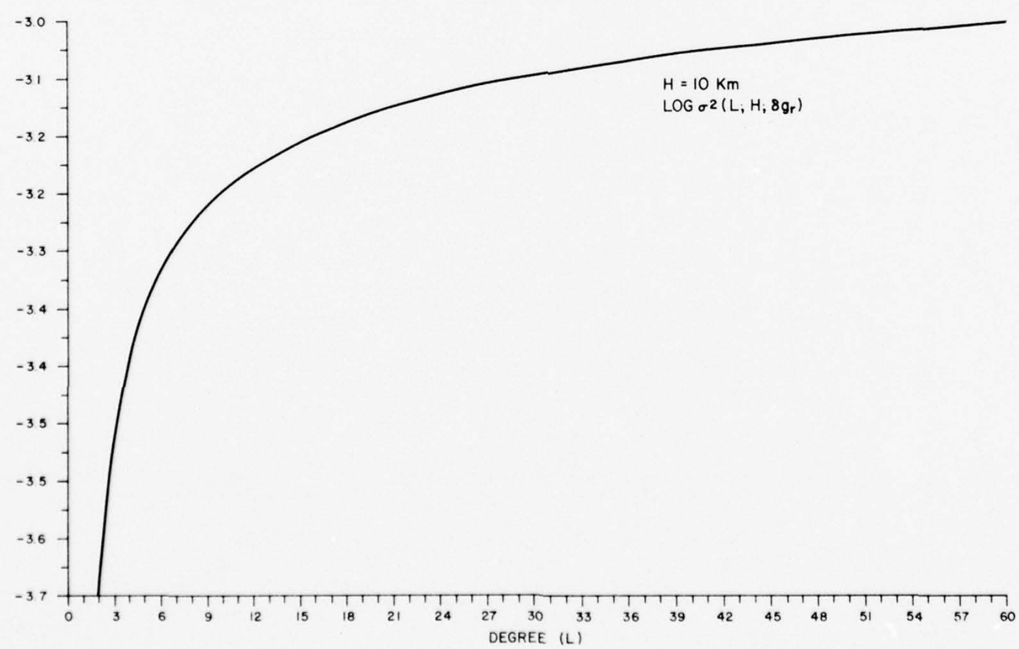

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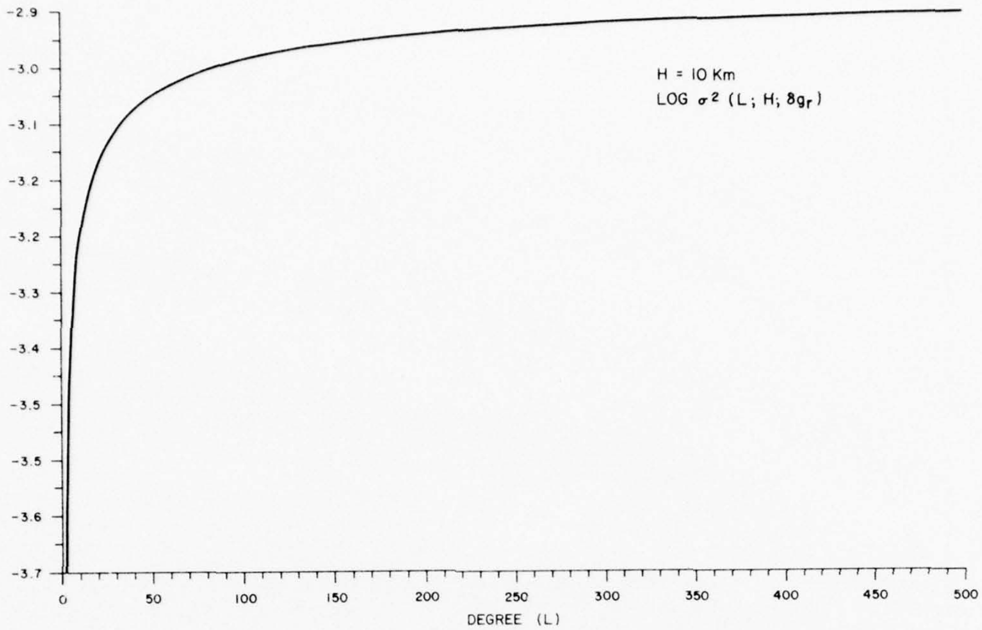
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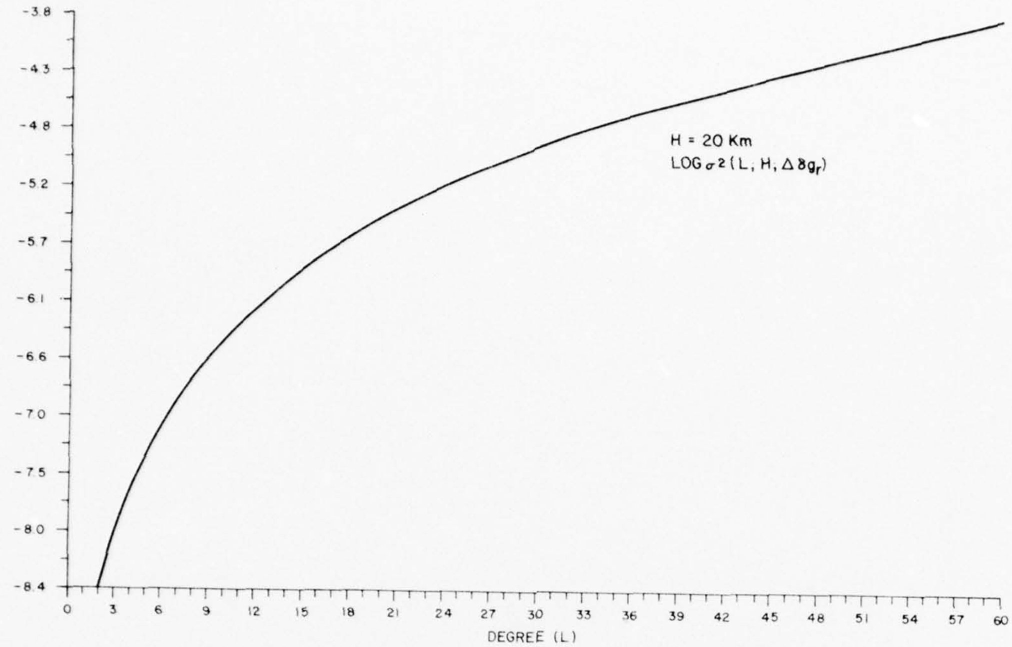
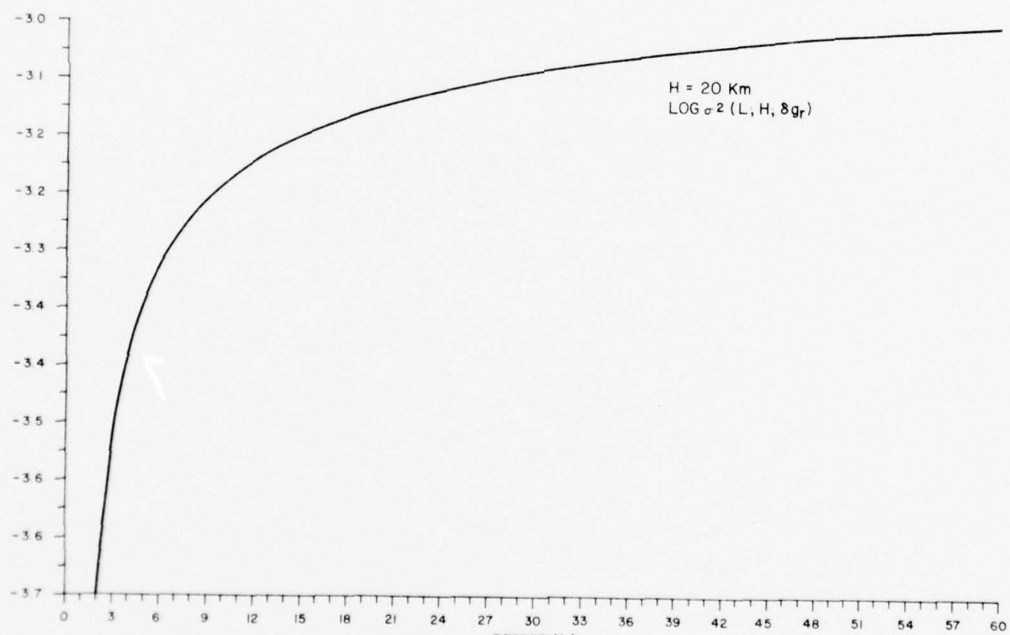
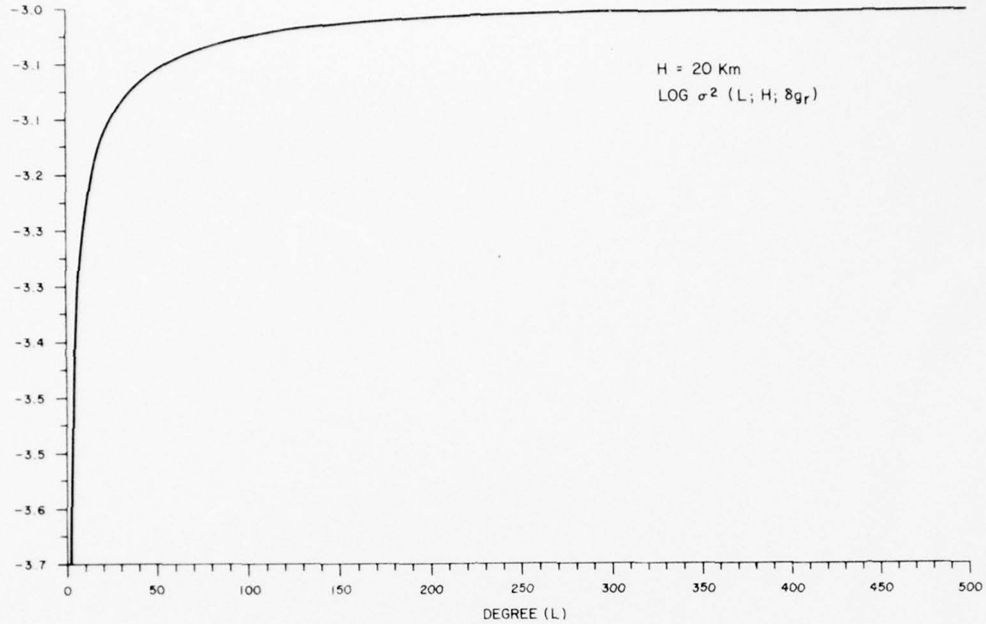
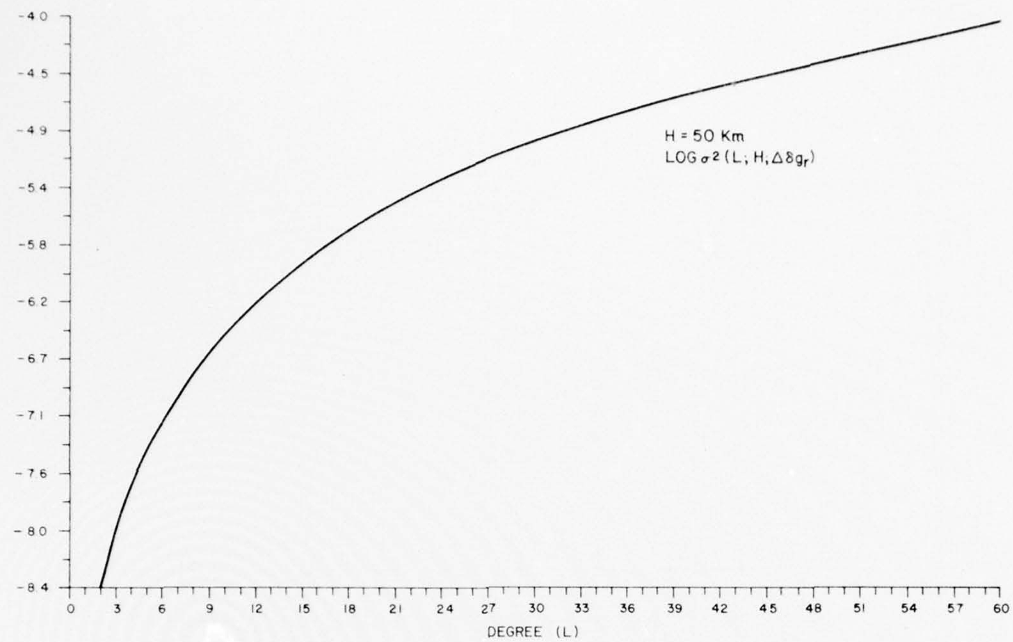
DEGREE (L)

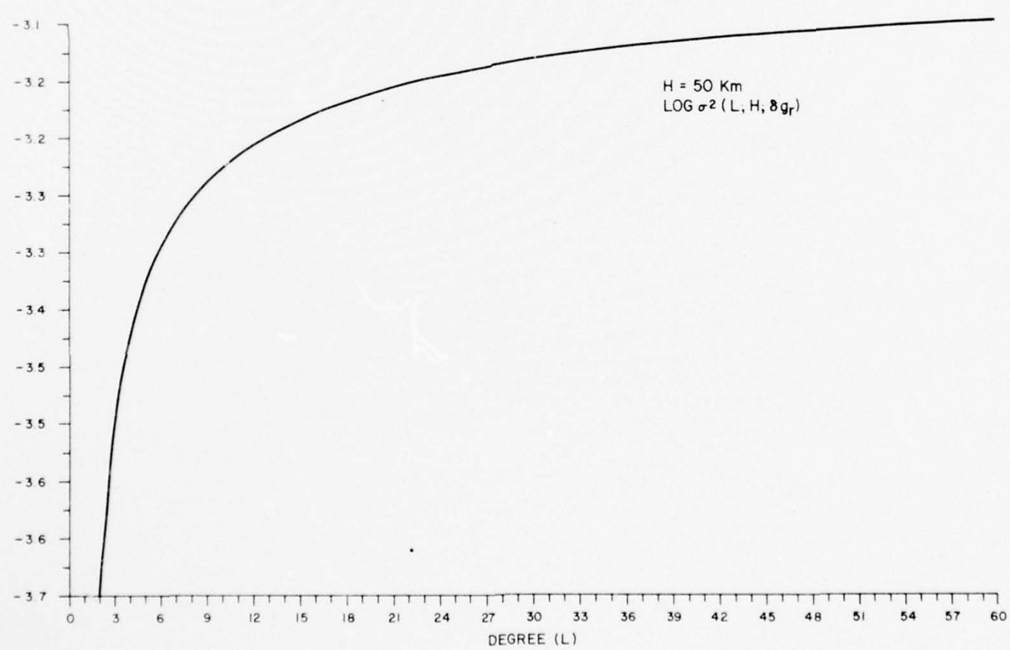

H = 20 Km
LOG σ2 (L, H, Δδgr)
DEGREE (L)

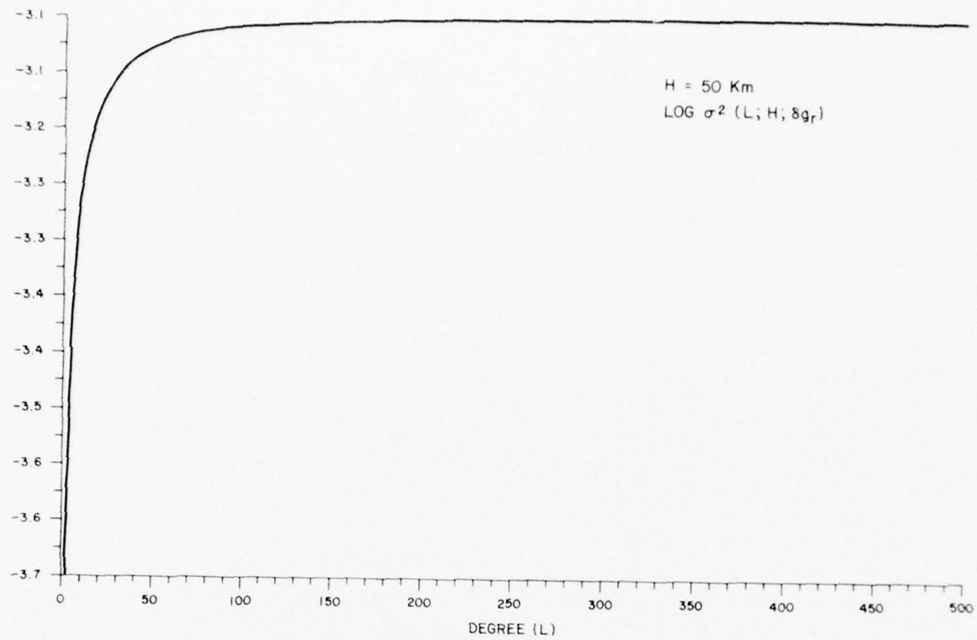

H = 20 Km
LOG σ2 (L, H, δgr)
DEGREE (L)

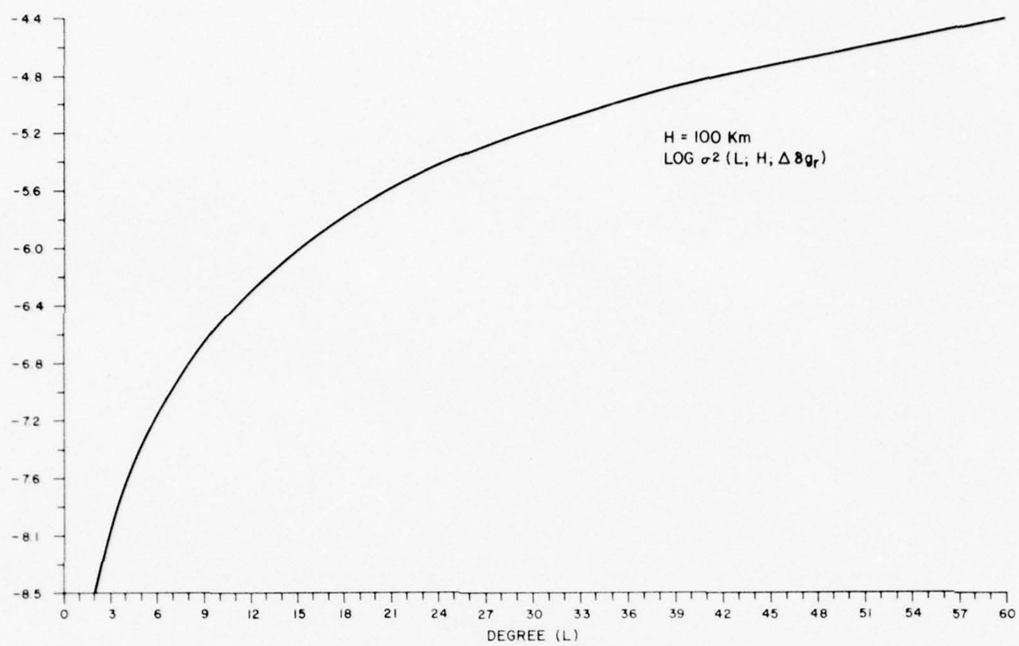


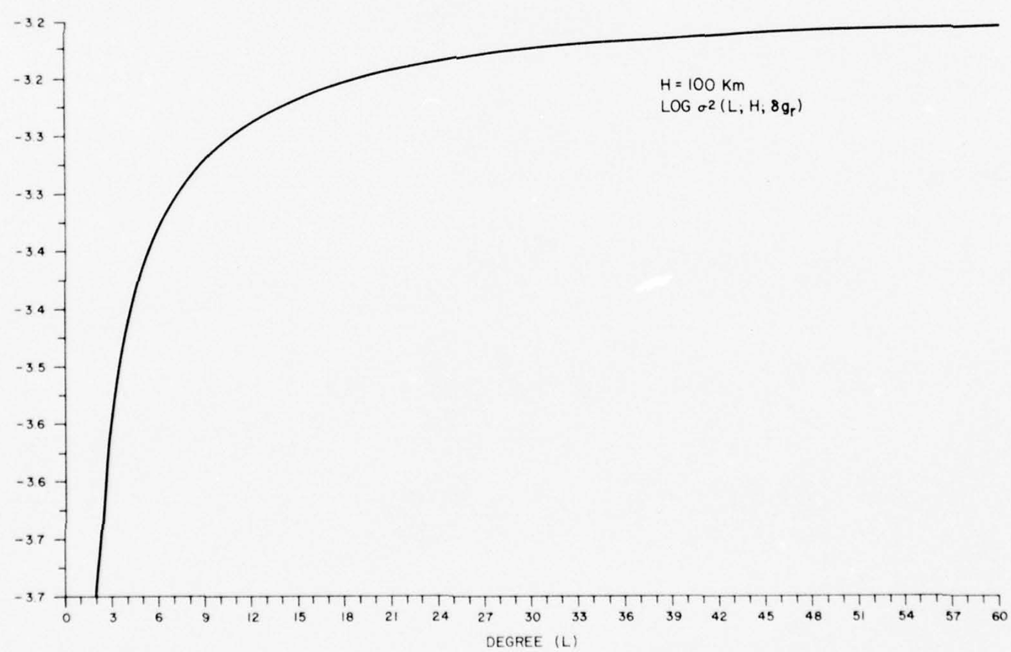
H = 20 Km
LOG σ2 (L; H; δgr)
DEGREE (L)
-3.0
-3.1
-3.1
-3.2
-3.3
-3.3
-3.4
-3.5
-3.5
-3.6
-3.7
0
50
100
150
200
250
300
350
400
450
500

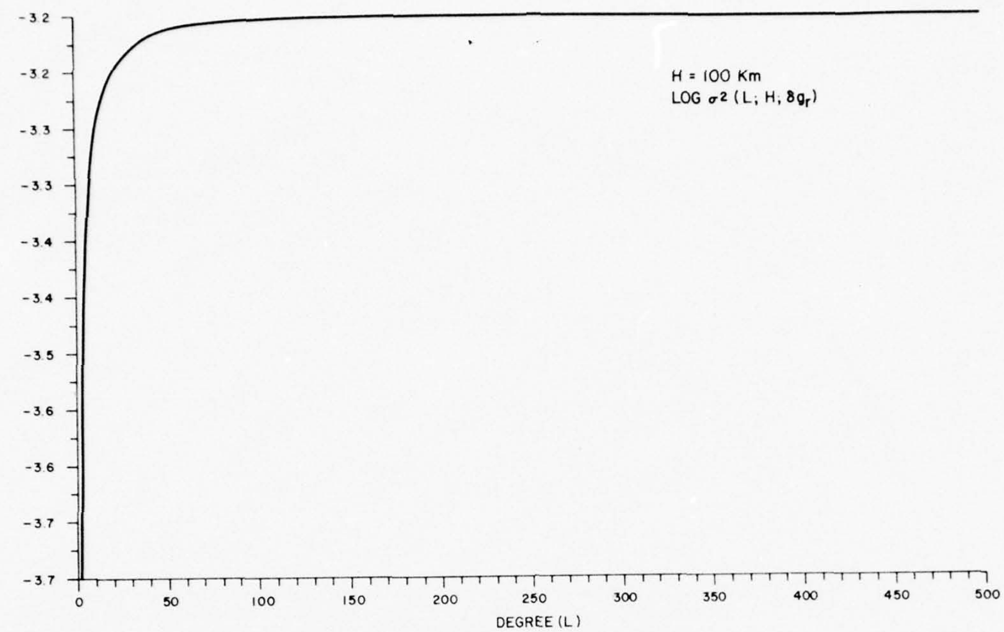

H = 50 Km
LOG σ2 (L, H, Δδgr)
DEGREE (L)

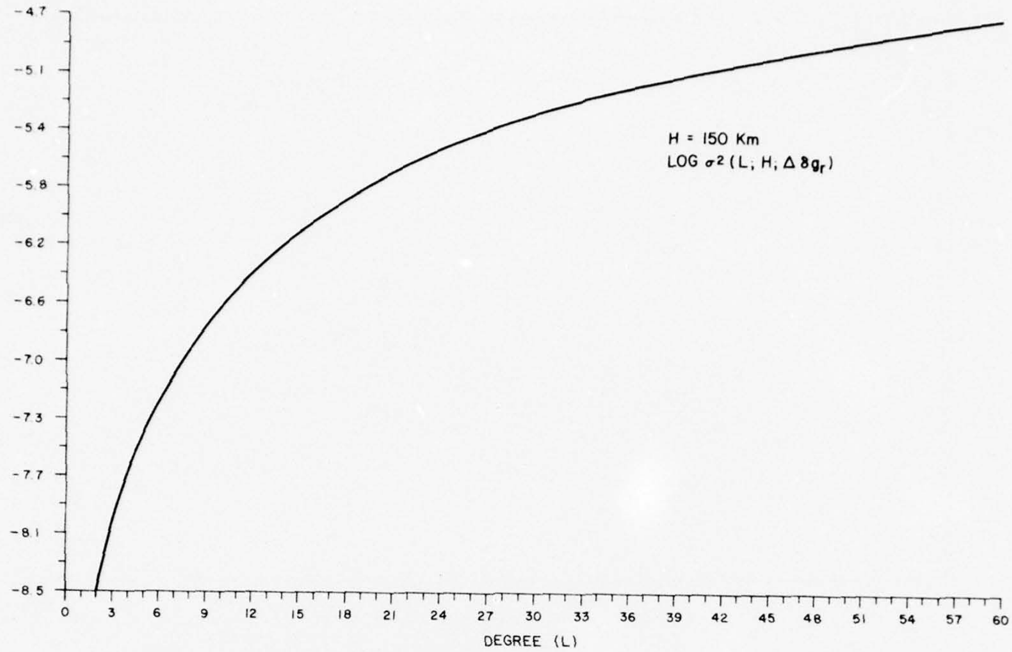

H = 50 Km
LOG σ2 (L, H, δgr)
DEGREE (L)

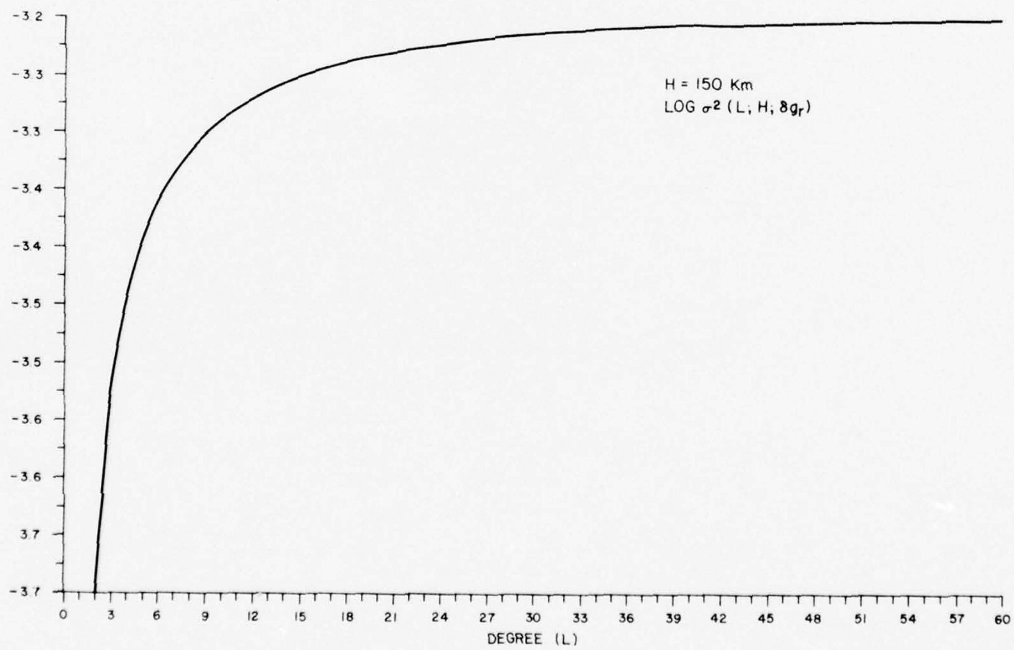

H = 50 Km
LOG σ^2 (L; H; δg_r)
-3.1
-3.1
-3.2
-3.3
-3.3
-3.4
-3.4
-3.5
-3.6
-3.6
-3.7
0
50
100
150
200
250
300
350
400
450
500
DEGREE (L)

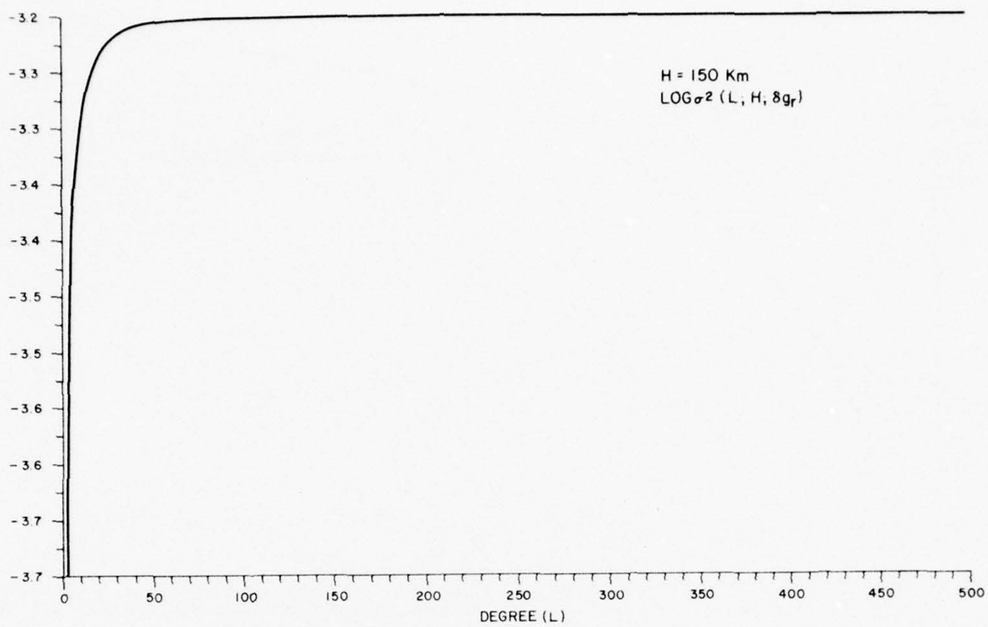

H = 100 Km
LOG σ^2(L; H; $\Delta\delta g_r$)
-4.4
-4.8
-5.2
-5.6
-6.0
-6.4
-6.8
-7.2
-7.6
-8.1
-8.5
0 3 6 9 12 15 18 21 24 27 30 33 36 39 42 45 48 51 54 57 60
DEGREE (L)

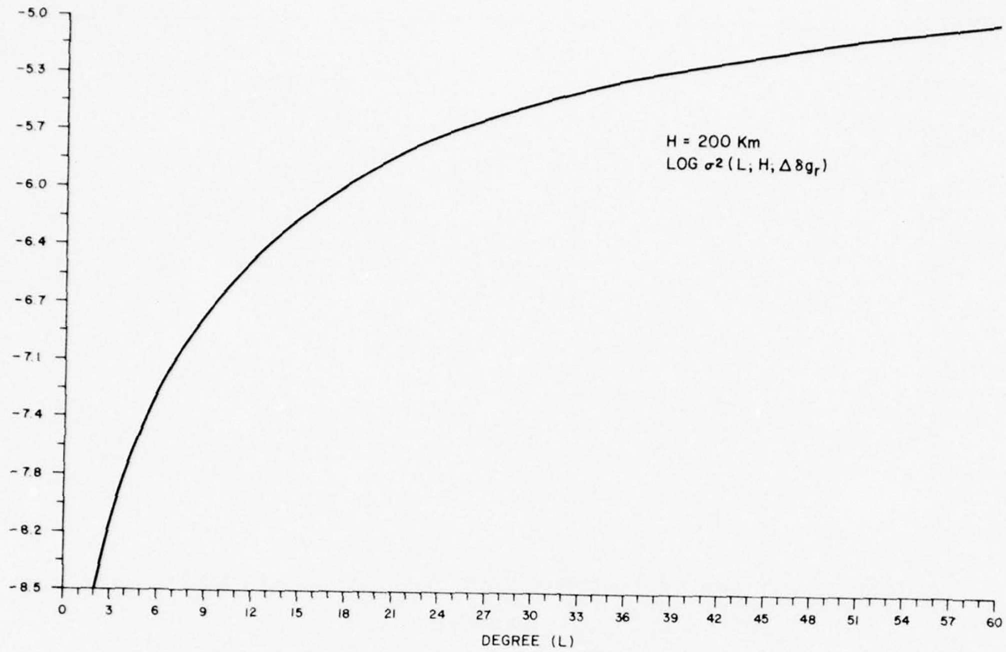

H = 100 Km
LOG σ^2(L; H; δg_r)
-3.2
-3.2
-3.3
-3.3
-3.4
-3.4
-3.5
-3.6
-3.6
-3.7
-3.7
0 3 6 9 12 15 18 21 24 27 30 33 36 39 42 45 48 51 54 57 60
DEGREE (L)

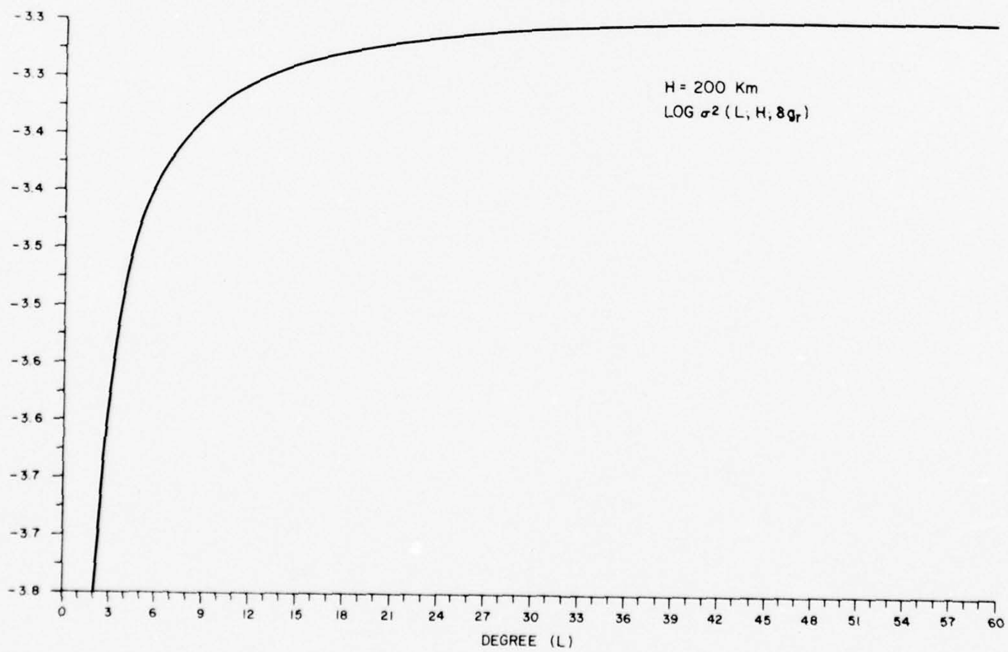

H = 100 Km
LOG σ2 (L; H; δgr)
-3.2
-3.2
-3.3
-3.3
-3.4
-3.4
-3.5
-3.6
-3.6
-3.7
-3.7
0
50
100
150
200
250
300
350
400
450
500
DEGREE (L)

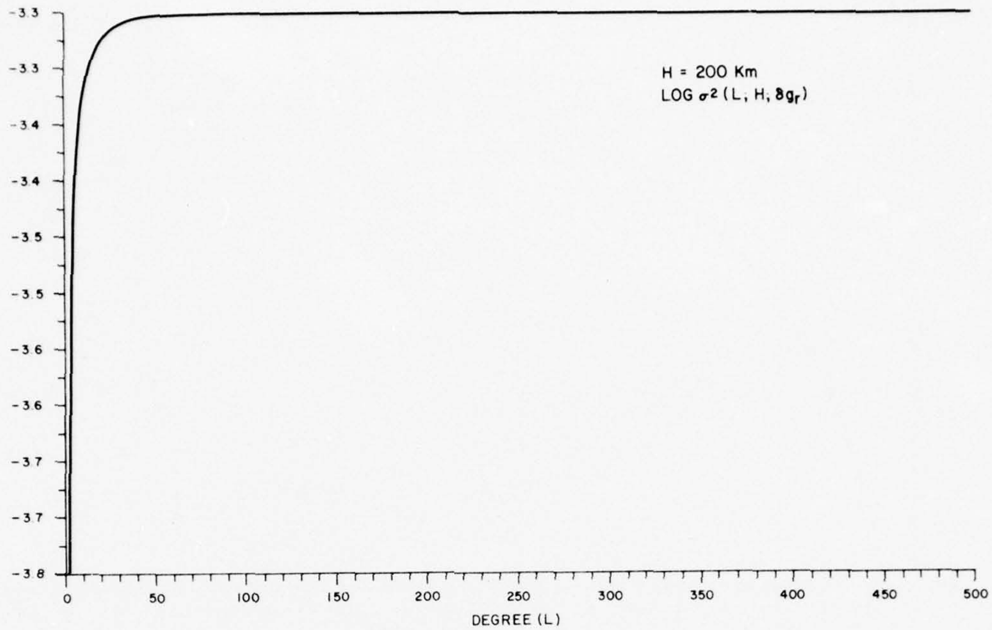

H = 150 Km
LOG σ2 (L; H; Δ δgr)
-4.7
-5.1
-5.4
-5.8
-6.2
-6.6
-7.0
-7.3
-7.7
-8.1
-8.5
0 3 6 9 12 15 18 21 24 27 30 33 36 39 42 45 48 51 54 57 60
DEGREE (L)


H = 150 Km
LOG σ2 (L; H; δgr)
-3.2
-3.3
-3.3
-3.4
-3.4
-3.5
-3.5
-3.6
-3.6
3.7
-3.7
0 3 6 9 12 15 18 21 24 27 30 33 36 39 42 45 48 51 54 57 60
DEGREE (L)

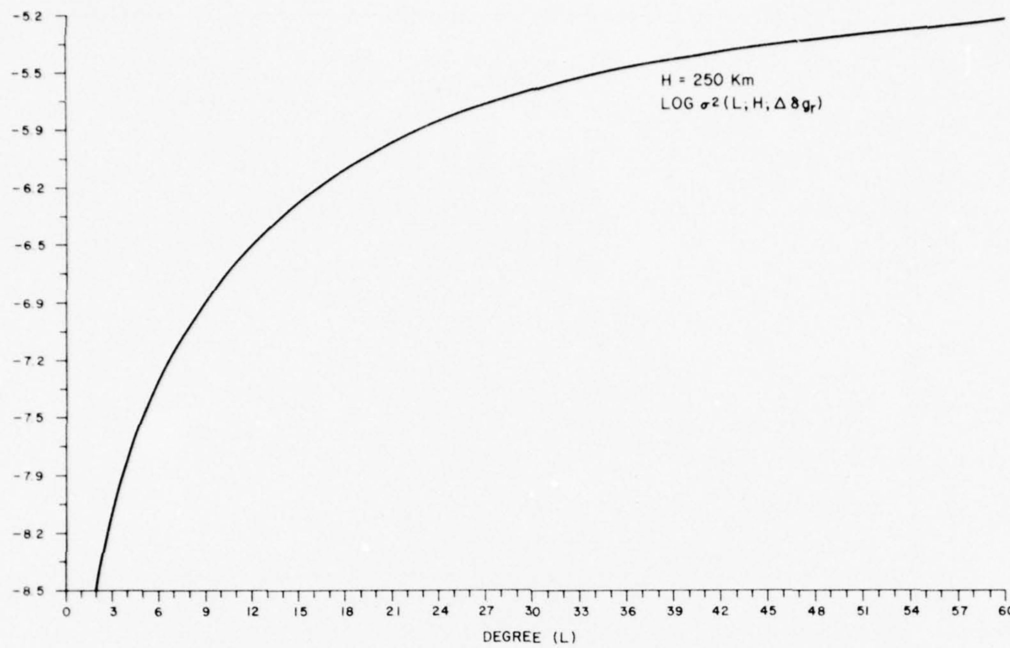

H = 150 Km
LOGσ2 (L, H, 8gr)
-3.2
-3.3
-3.3
-3.4
-3.4
-3.5
-3.5
-3.6
-3.6
-3.7
-3.7
0
50
100
150
200
250
300
350
400
450
500
DEGREE (L)

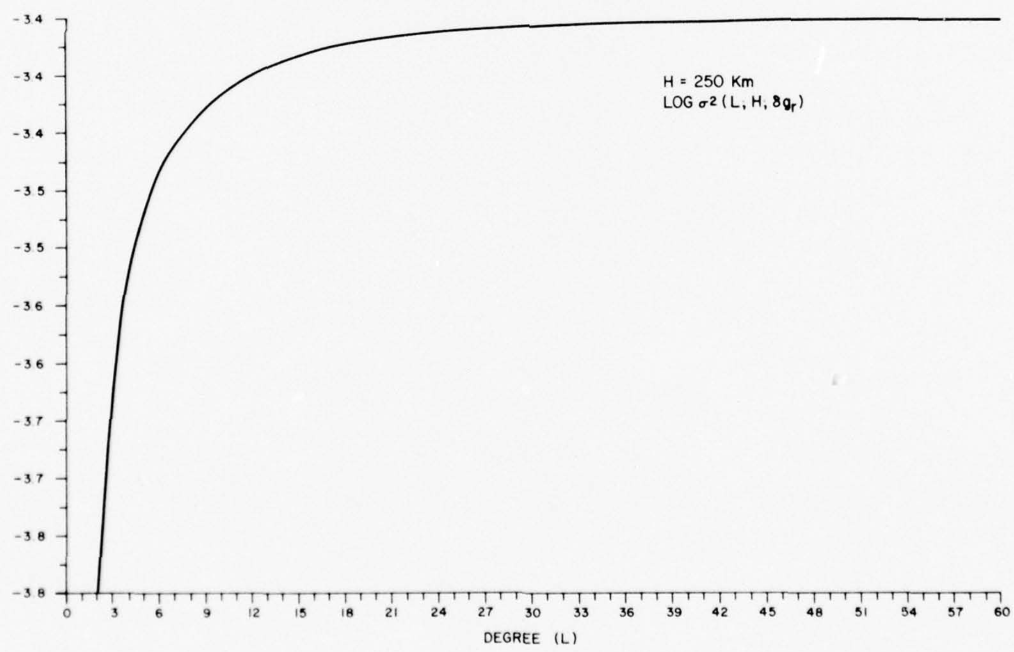

-5.0
-5.3
-5.7
-6.0
-6.4
-6.7
-7.1
-7.4
-7.8
-8.2
-8.5
0 3 6 9 12 15 18 21 24 27 30 33 36 39 42 45 48 51 54 57 60
DEGREE (L)
H = 200 Km
LOG σ2 (L; H; Δδgr)

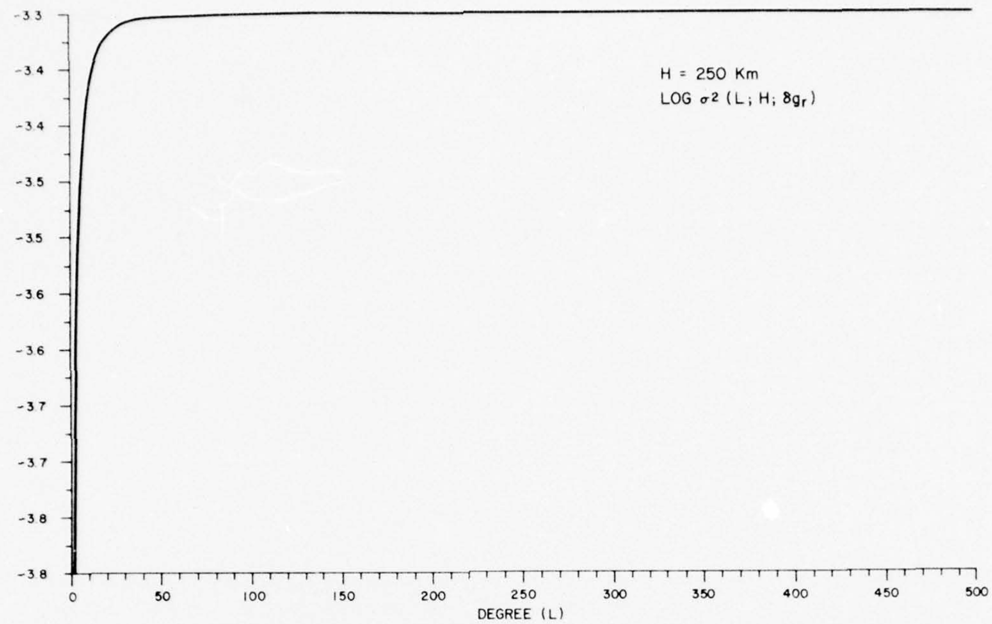

-3.3
-3.3
-3.4
-3.4
-3.5
-3.5
-3.6
-3.6
-3.7
-3.7
-3.8
0 3 6 9 12 15 18 21 24 27 30 33 36 39 42 45 48 51 54 57 60
DEGREE (L)
H = 200 Km
LOG σ2 (L; H; δgr)



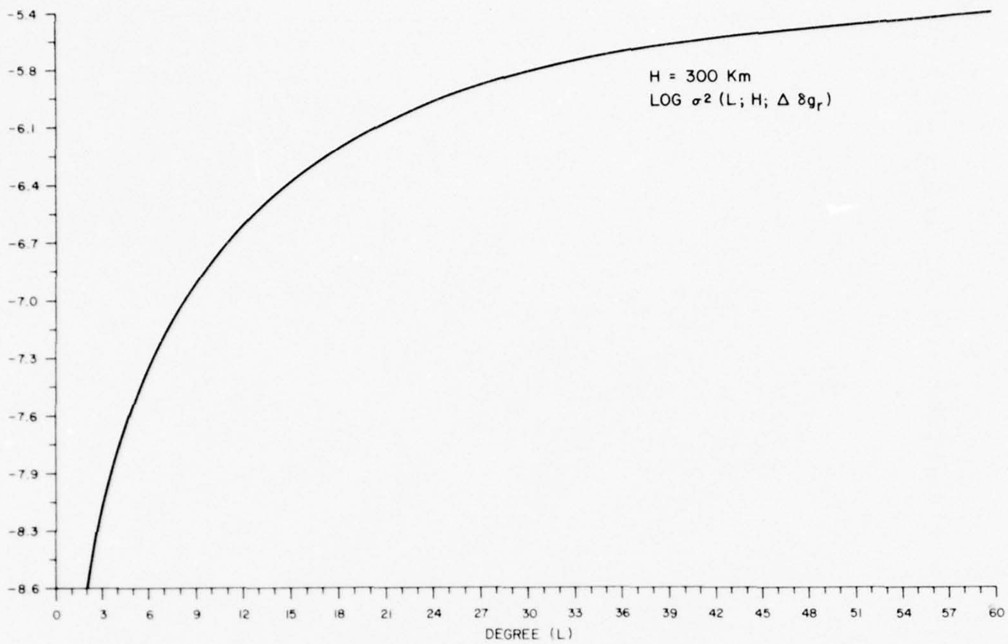
H = 200 Km
LOG σ2 (L, H, δgr)
-3.3
-3.3
-3.4
-3.4
-3.5
-3.5
-3.6
-3.6
-3.7
-3.7
-3.8
0
50
100
150
200
250
300
350
400
450
500
DEGREE (L)

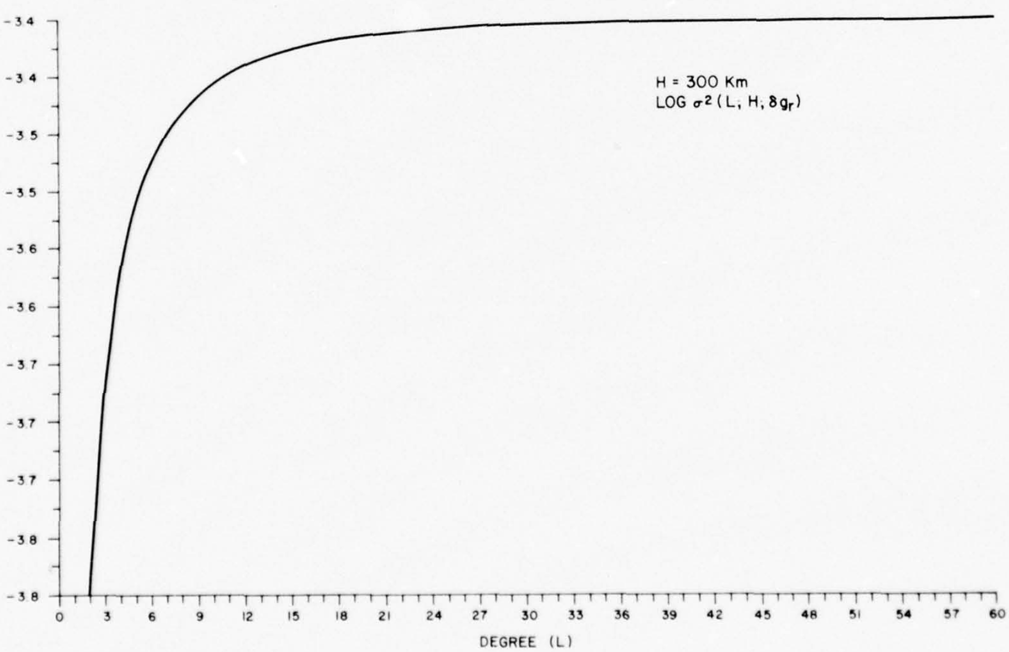

H = 250 Km
LOG $\sigma^2(L, H, \Delta\delta g_r)$
DEGREE (L)

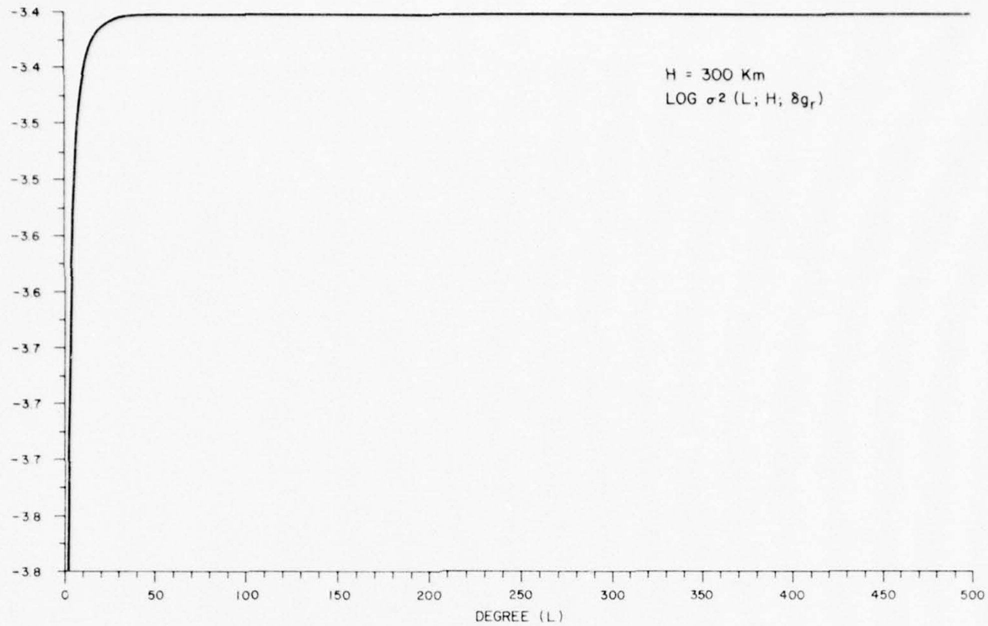

H = 250 Km
LOG $\sigma^2(L, H, \delta g_r)$
DEGREE (L)

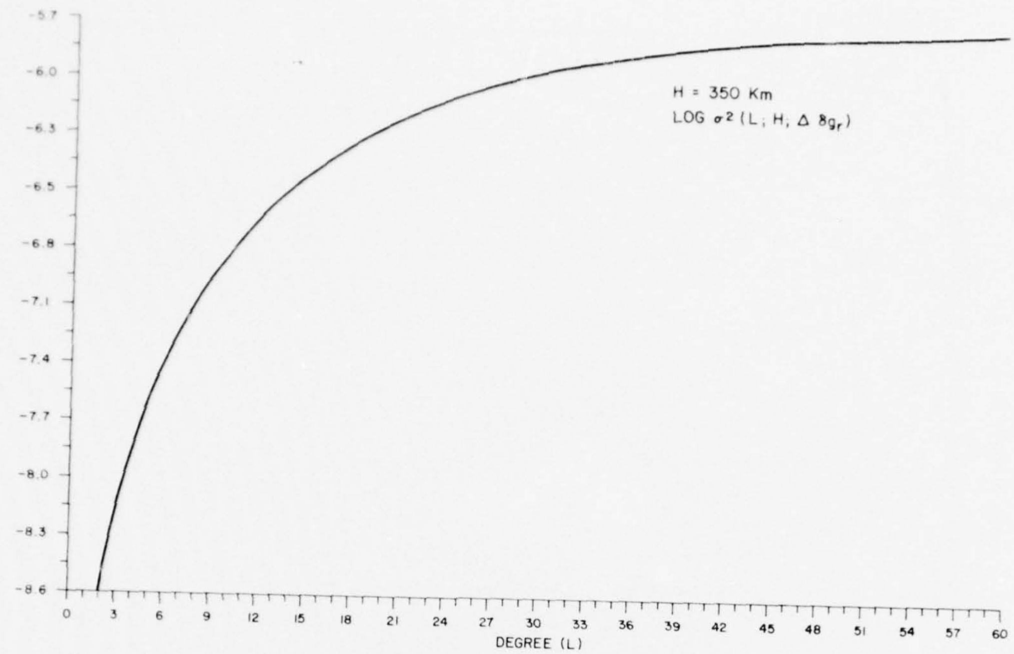


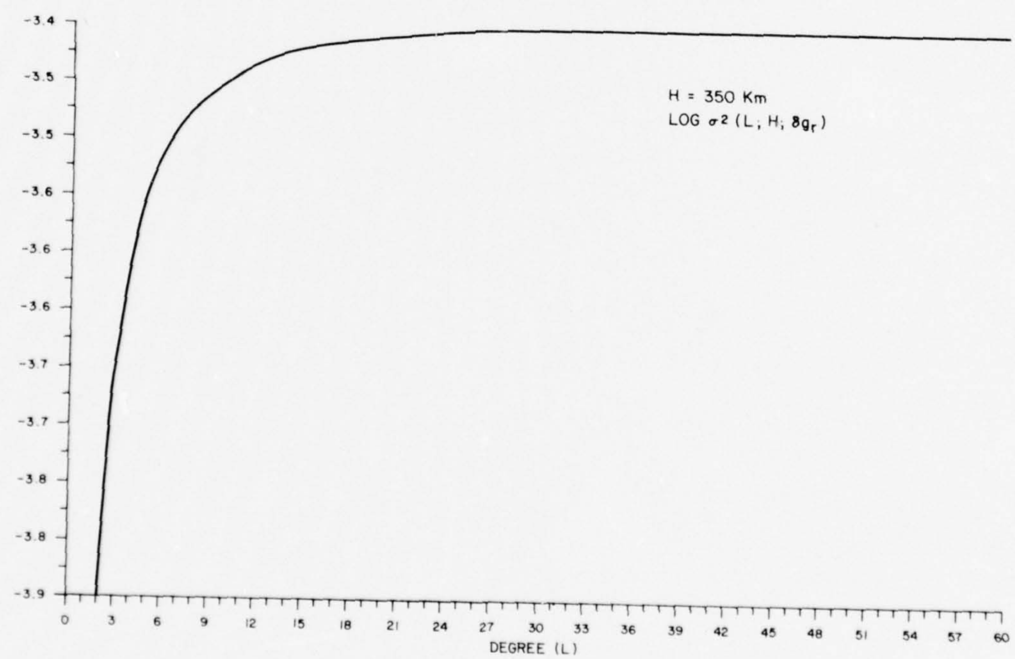
H = 250 Km
LOG σ2 (L; H; δgr)
-3.3
-3.4
-3.4
-3.5
-3.5
-3.6
-3.6
-3.7
-3.7
-3.8
-3.8
0
50
100
150
200
250
300
350
400
450
500
DEGREE (L)

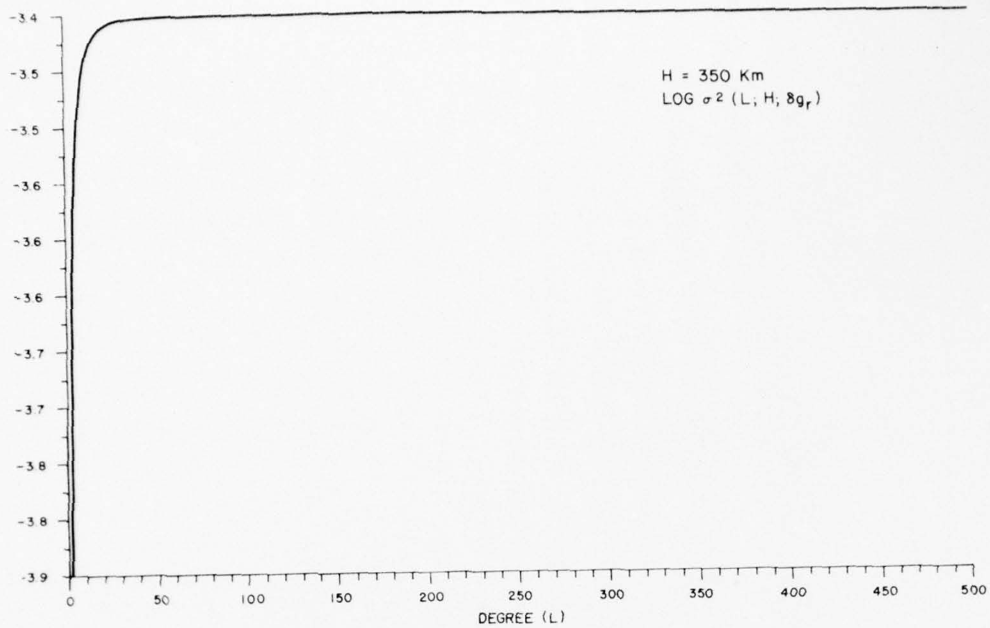

H = 300 Km
LOG σ^2 (L; H; Δ δg_r)
DEGREE (L)


H = 300 Km
LOG σ^2 (L, H, δg_r)
DEGREE (L)

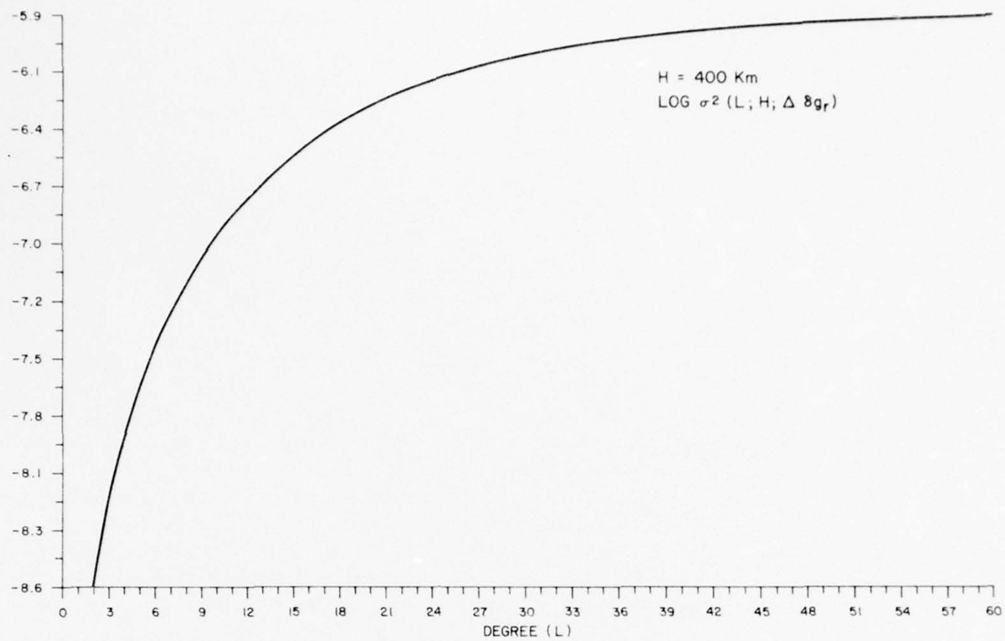

H = 300 Km
LOG σ2 (L; H; δgr)
-3.4
-3.4
-3.5
-3.5
-3.6
-3.6
-3.7
-3.7
-3.7
-3.8
-3.8
0
50
100
150
200
250
300
350
400
450
500
DEGREE (L)


H = 350 Km
LOG σ2 (L; H; Δ δgr)
DEGREE (L)

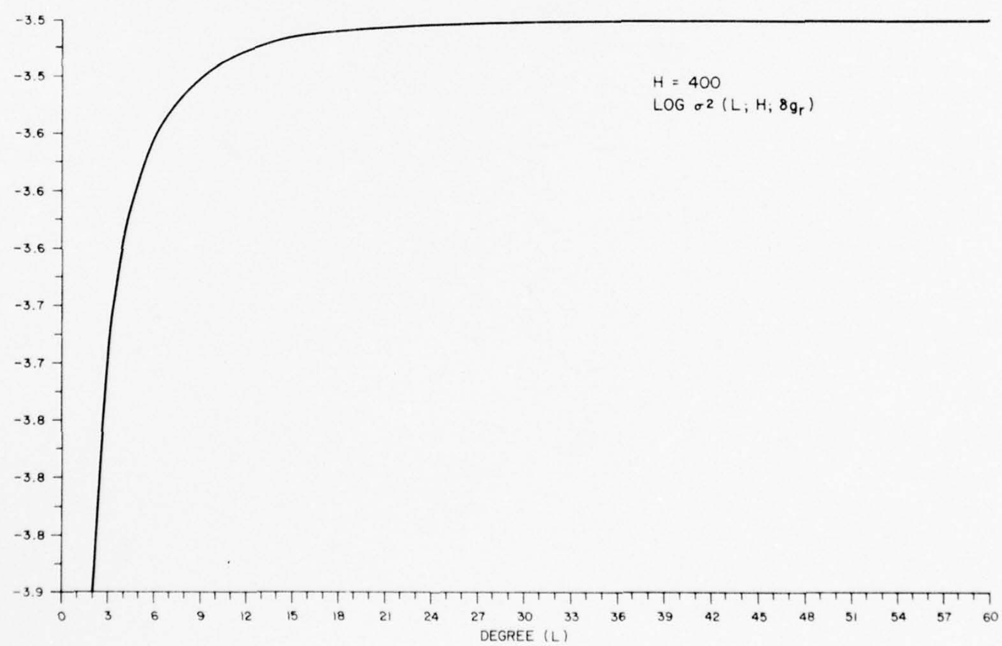

H = 350 Km
LOG σ2 (L; H; δgr)
DEGREE (L)



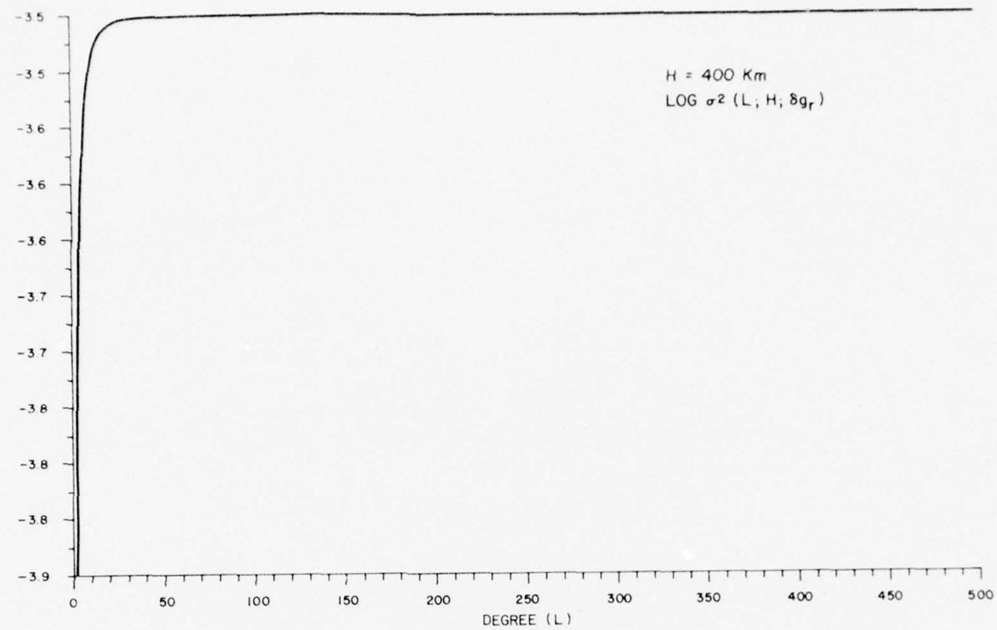
H = 350 Km
LOG σ 2 (L; H; δg r)
-3.4
-3.5
-3.5
-3.6
-3.6
-3.6
-3.7
-3.7
-3.8
-3.8
-3.9
0
50
100
150
200
250
300
350
400
450
500
DEGREE (L)

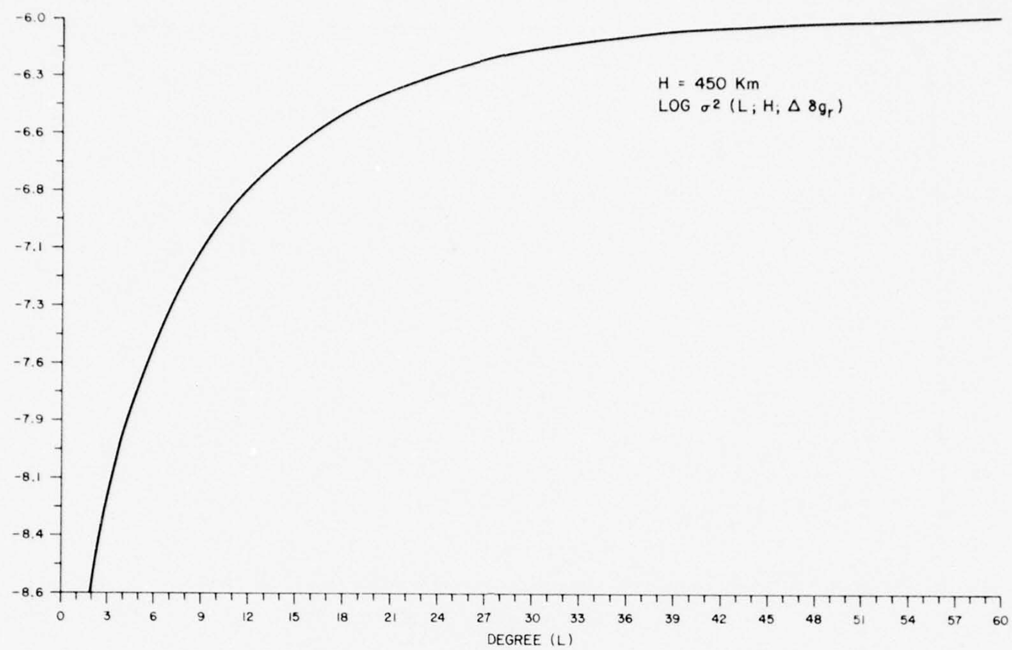


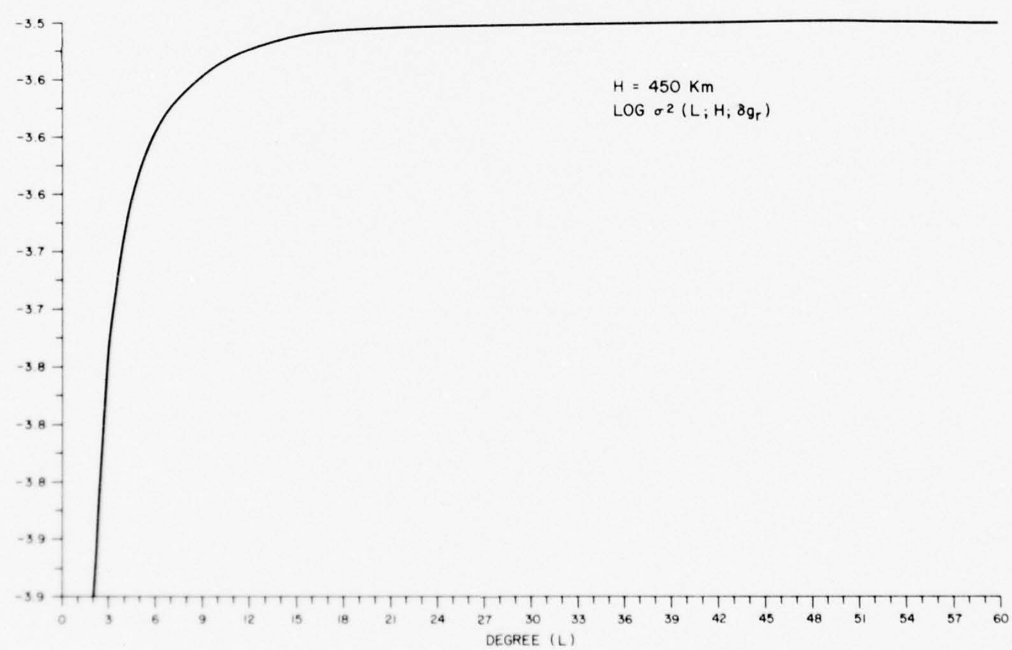
H = 400 Km
LOG σ2 (L; H; Δ δgr)
-5.9
-6.1
-6.4
-6.7
-7.0
-7.2
-7.5
-7.8
-8.1
-8.3
-8.6
0 3 6 9 12 15 18 21 24 27 30 33 36 39 42 45 48 51 54 57 60
DEGREE (L)

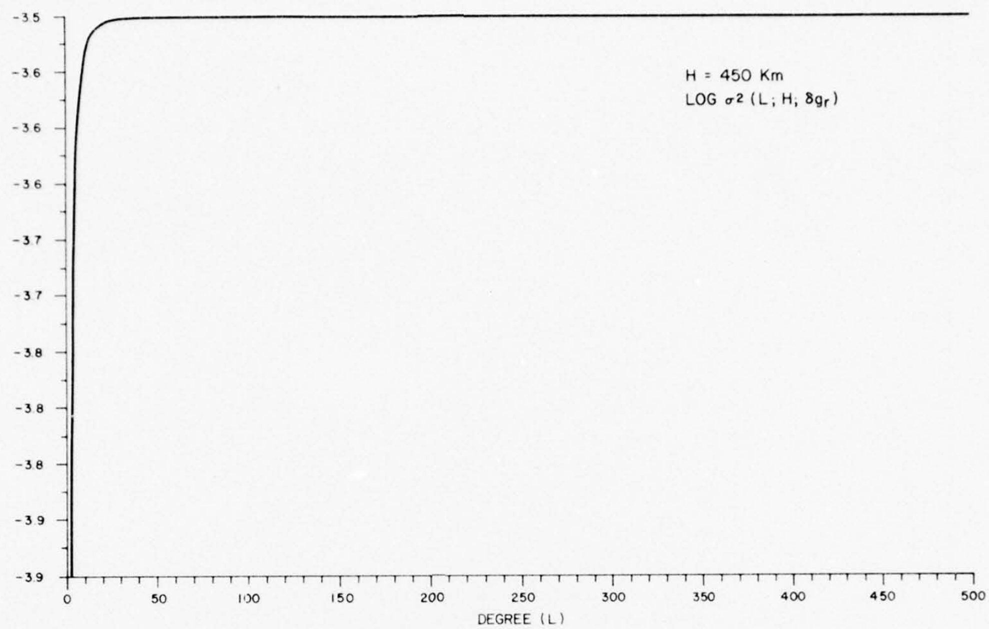


H = 400
LOG σ2 (L; H; δgr)
-3.5
-3.5
-3.6
-3.6
-3.6
-3.7
-3.7
-3.8
-3.8
-3.8
-3.9
0 3 6 9 12 15 18 21 24 27 30 33 36 39 42 45 48 51 54 57 60
DEGREE (L)

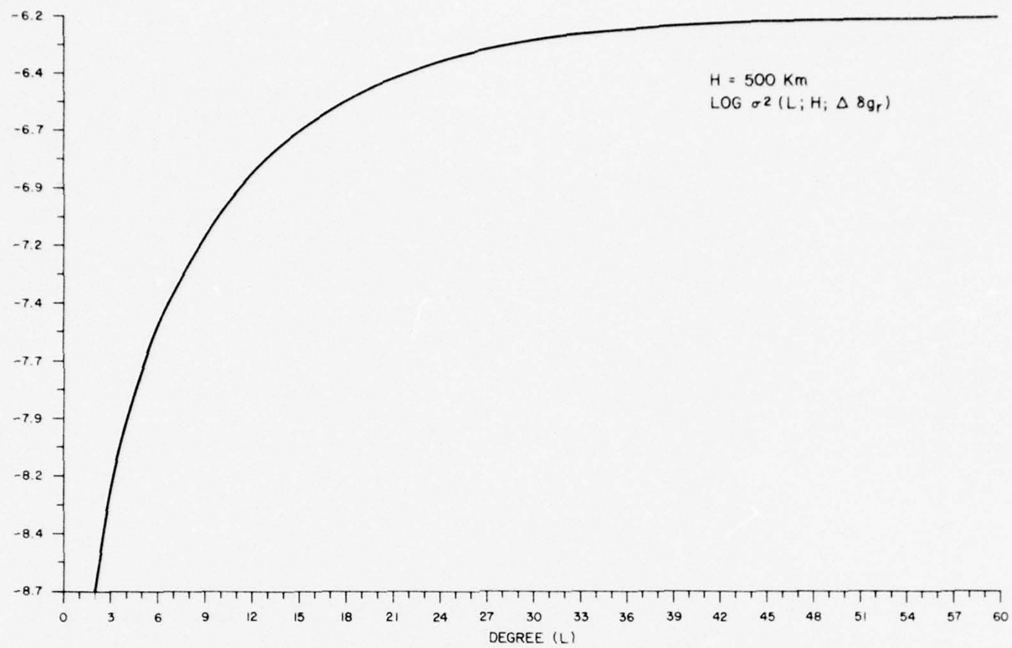

H = 400 Km
LOG σ2 (L; H; δgr)
-3.5
-3.5
-3.6
-3.6
-3.6
-3.7
-3.7
-3.8
-3.8
-3.8
-3.9
0
50
100
150
200
250
300
350
400
450
500
DEGREE (L)

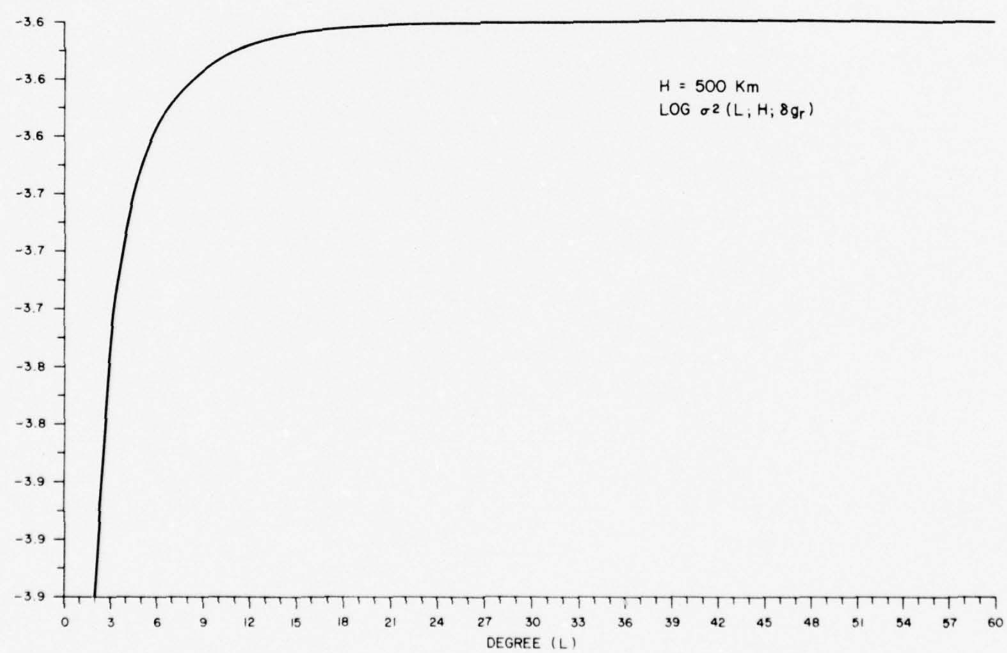

H = 450 Km
LOG σ^2 (L; H; $\Delta\,\delta g_r$)
DEGREE (L)

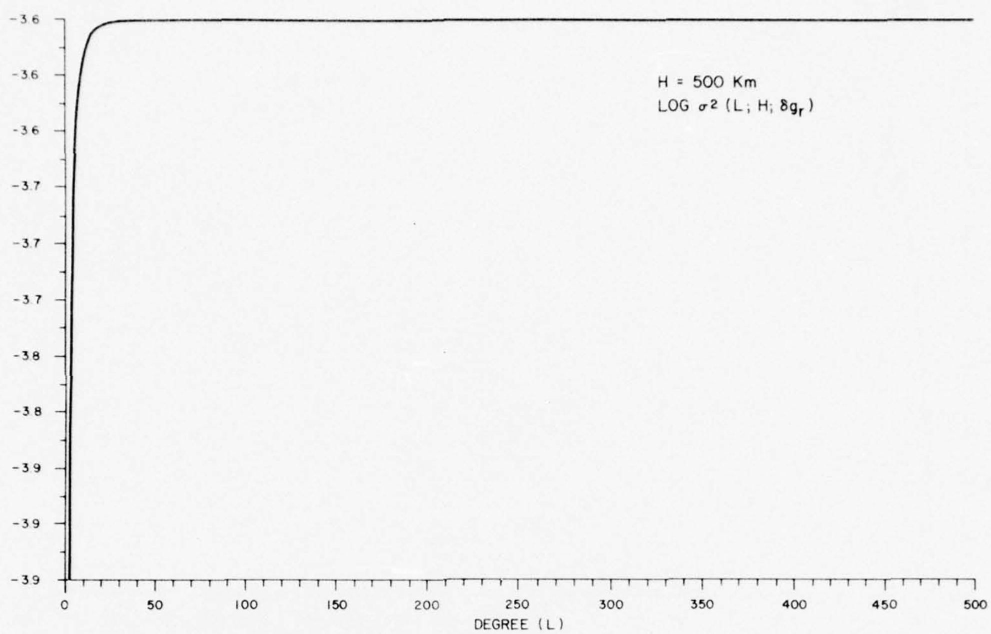

H = 450 Km
LOG σ^2 (L; H; δg_r)
DEGREE (L)

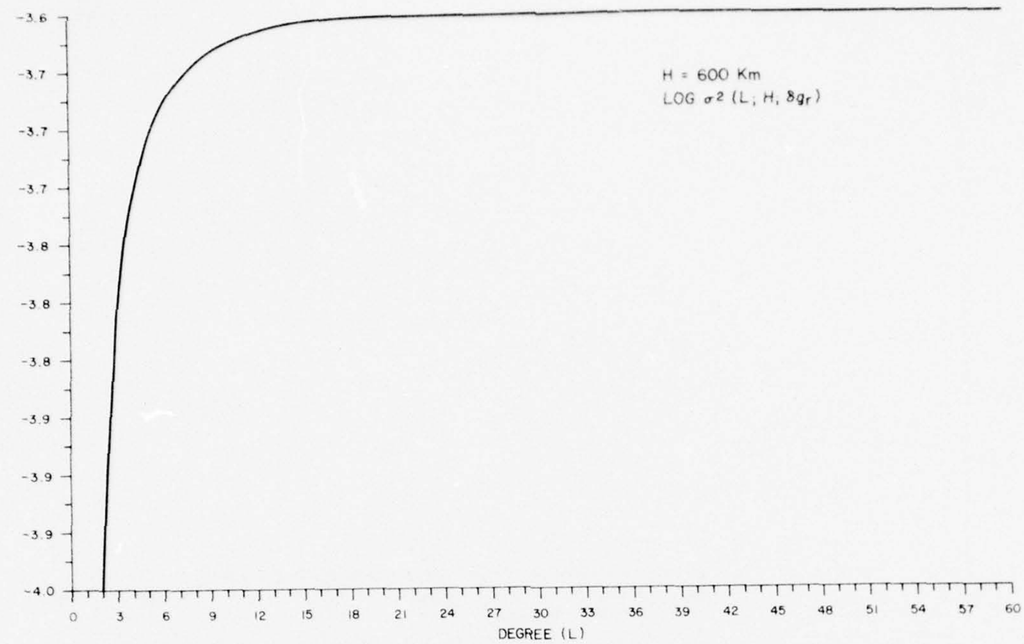


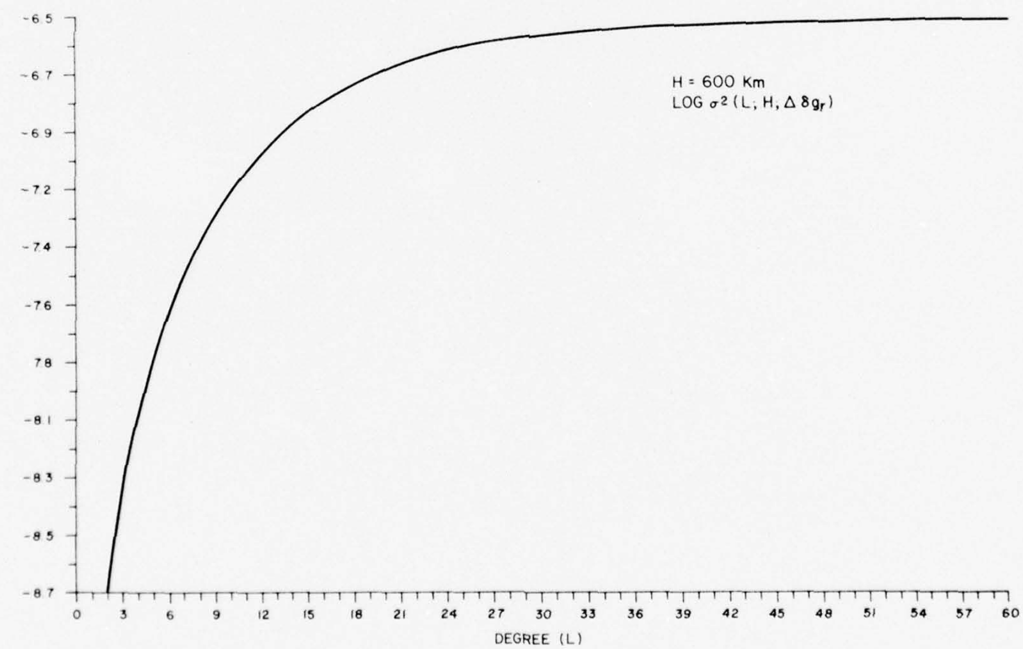
H = 450 Km
LOG σ2 (L; H; δgr)
-3.5
-3.6
-3.6
-3.6
-3.7
-3.7
-3.8
-3.8
-3.8
-3.9
-3.9
0
50
100
150
200
250
300
350
400
450
500
DEGREE (L)

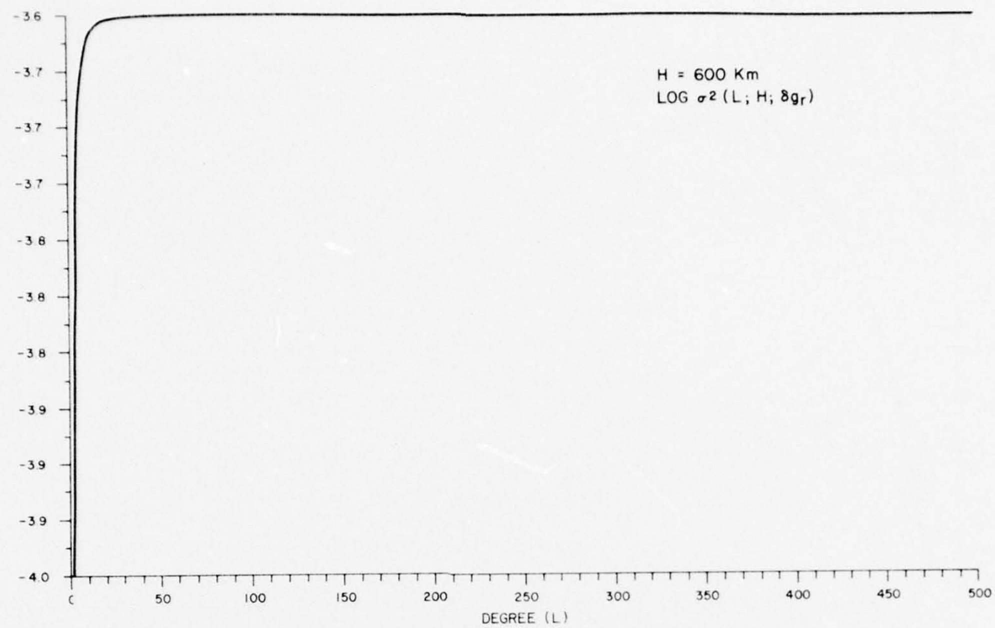

H = 500 Km
LOG σ2 (L; H; Δ δgr)
DEGREE (L)


H = 500 Km
LOG σ2 (L; H; δgr)
DEGREE (L)

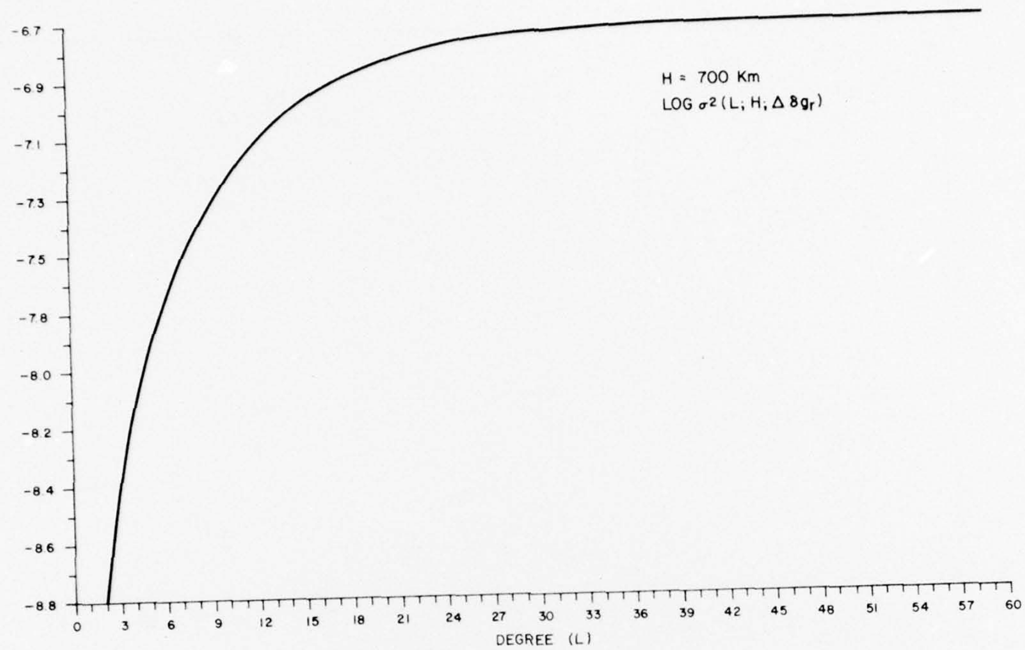

H = 500 Km
LOG σ^2 (L; H; δg_r)
-3.6
-3.6
-3.6
-3.7
-3.7
-3.7
-3.8
-3.8
-3.9
-3.9
-3.9
0
50
100
150
200
250
300
350
400
450
500
DEGREE (L)


H = 600 Km
LOG σ^2 (L; H, δg_r)
DEGREE (L)

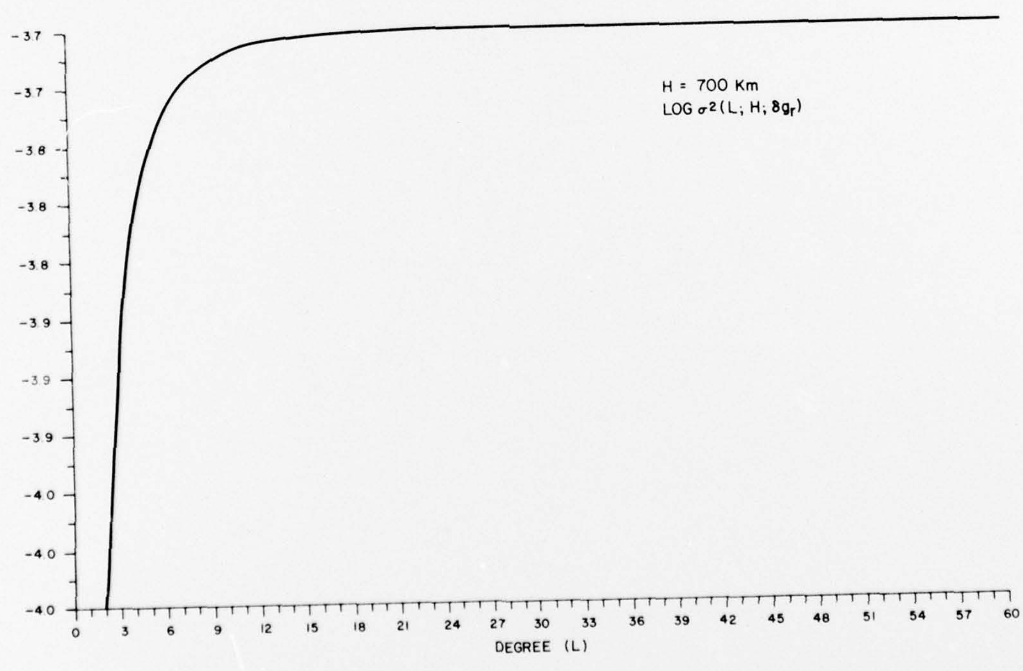

H = 600 Km
LOG σ^2 (L; H, $\Delta \delta g_r$)
DEGREE (L)



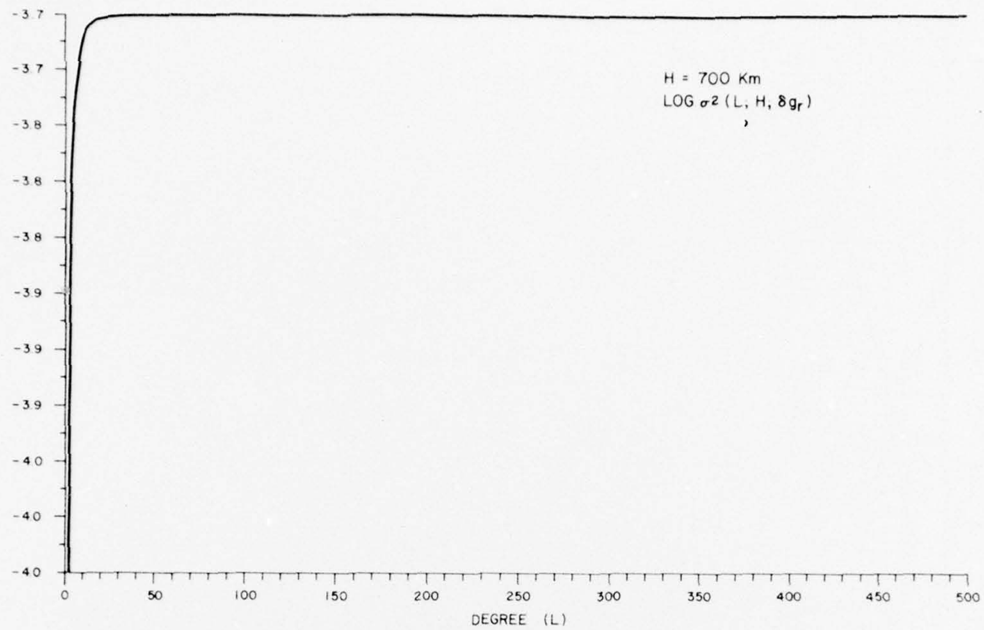
H = 600 Km
LOG σ2 (L; H; δgr)
-3.6
-3.7
-3.7
-3.7
-3.8
-3.8
-3.8
-3.9
-3.9
-3.9
-4.0
0
50
100
150
200
250
300
350
400
450
500
DEGREE (L)

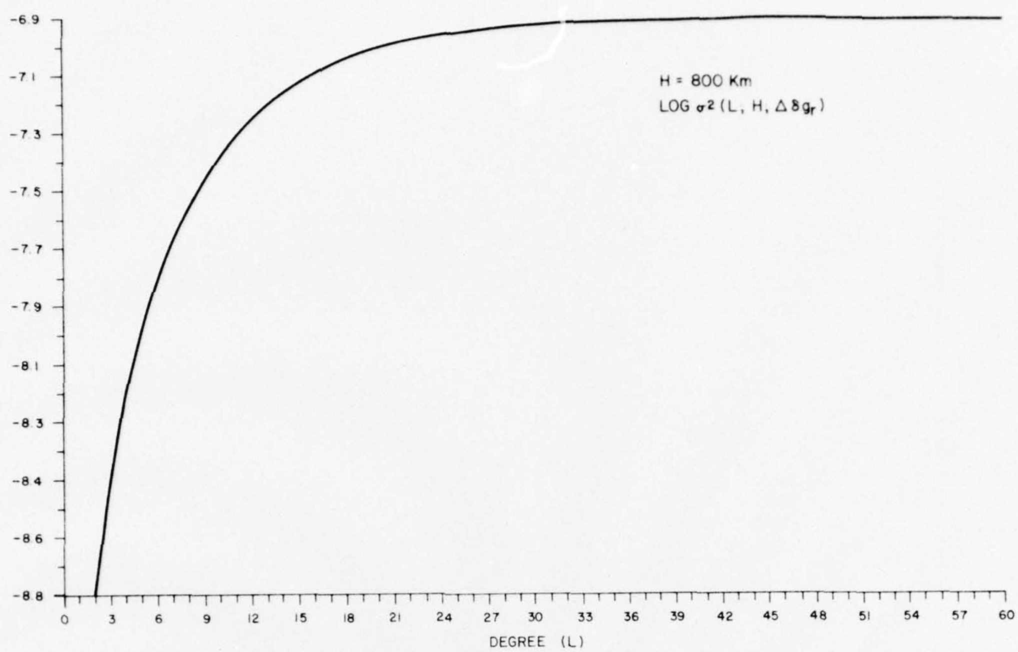


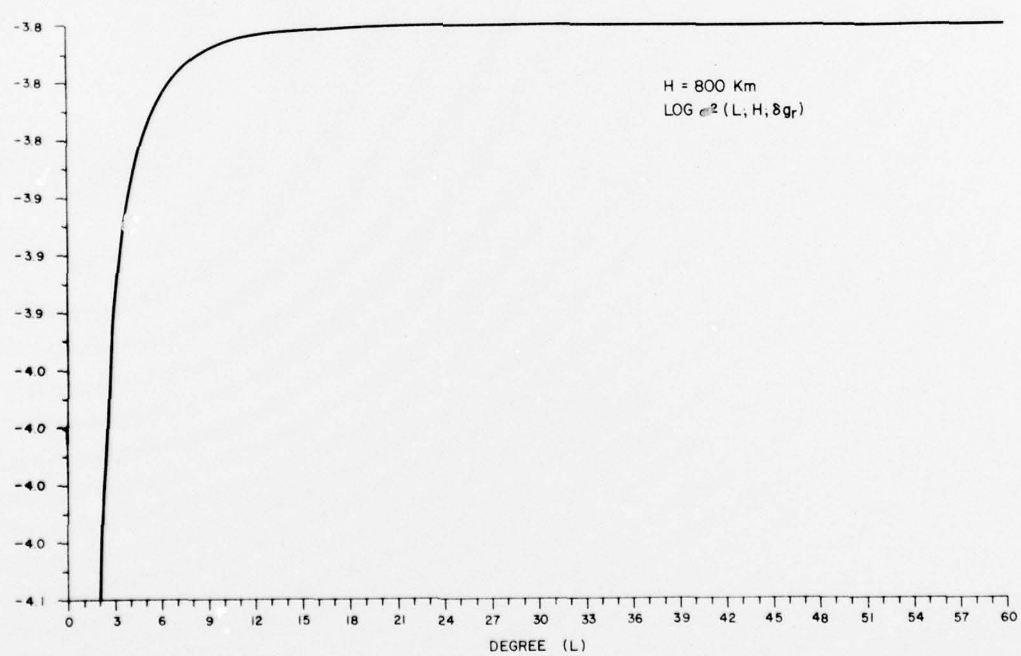
H = 700 Km
LOG σ^2(L; H; $\Delta\delta g_r$)
-6.7
-6.9
-7.1
-7.3
-7.5
-7.8
-8.0
-8.2
-8.4
-8.6
-8.8
0 3 6 9 12 15 18 21 24 27 30 33 36 39 42 45 48 51 54 57 60
DEGREE (L)

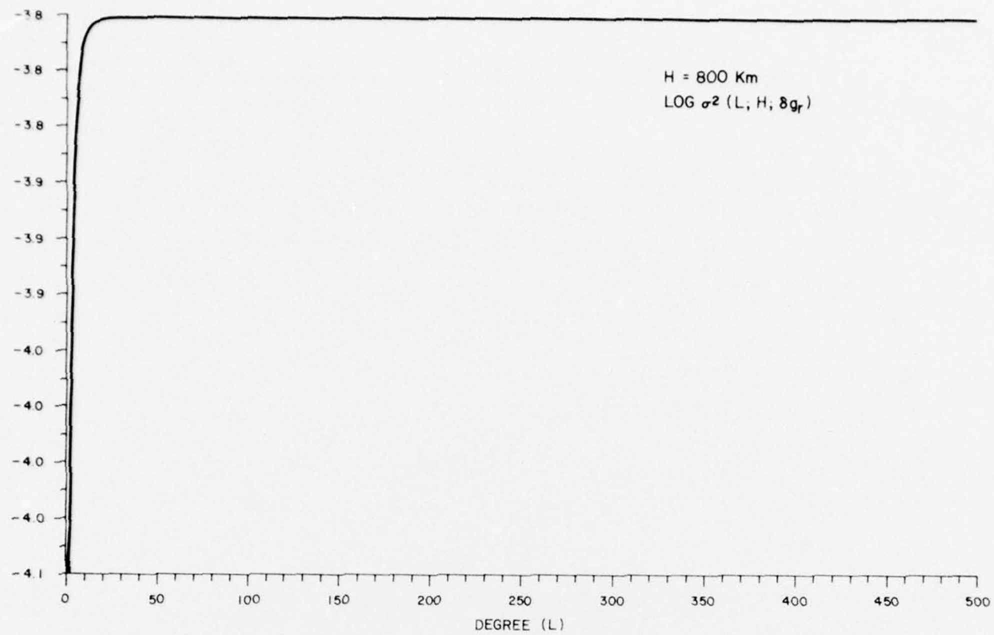


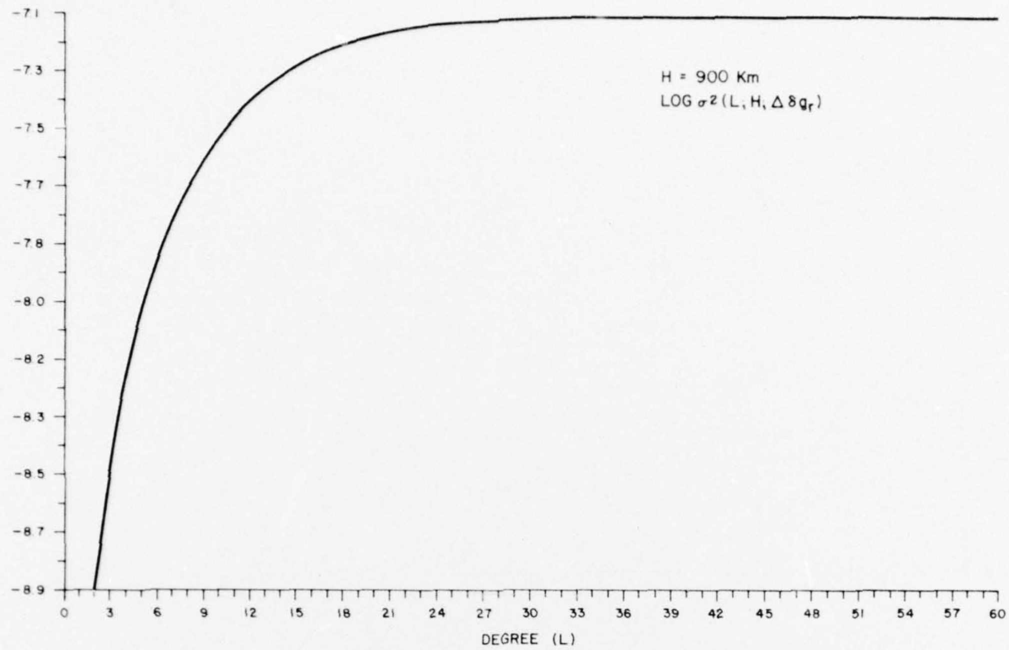
H = 700 Km
LOG σ^2(L; H; δg_r)
-3.7
-3.7
-3.8
-3.8
-3.8
-3.9
-3.9
-3.9
-4.0
-4.0
-4.0
0 3 6 9 12 15 18 21 24 27 30 33 36 39 42 45 48 51 54 57 60
DEGREE (L)

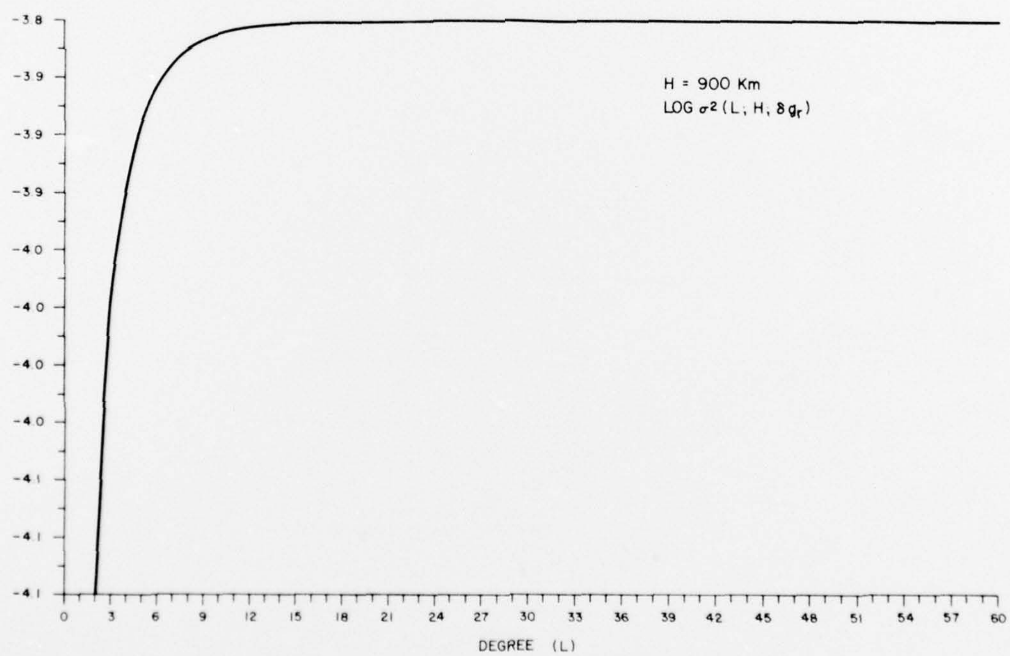

H = 700 Km
LOG σ^2 (L, H, δg_r)
DEGREE (L)

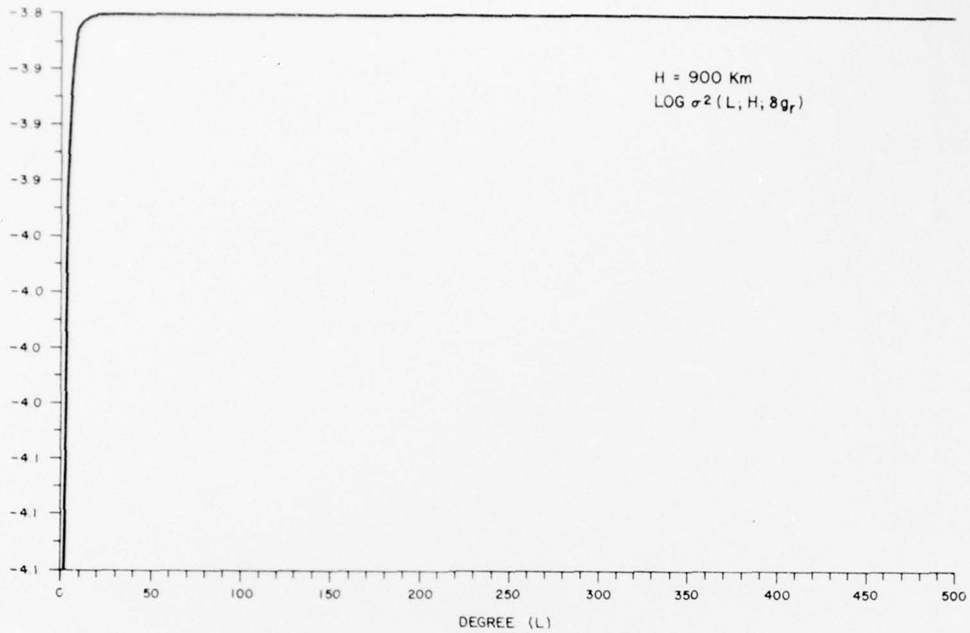

H = 800 Km
LOG σ^2 (L, H, $\Delta\delta g_r$)
DEGREE (L)

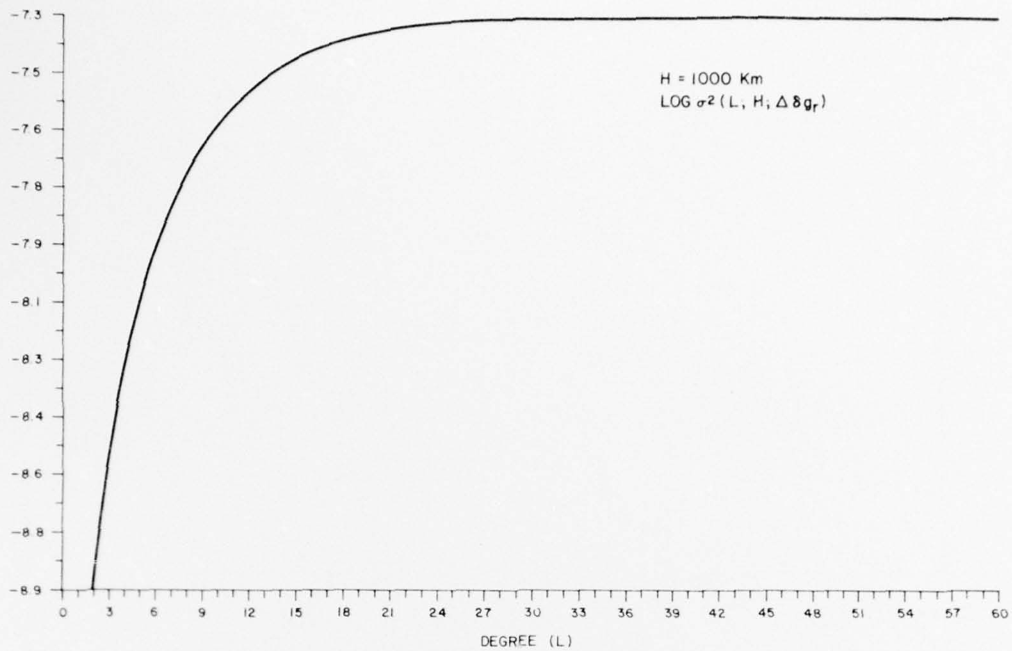

H = 800 Km
LOG σ^2 (L, H, δg_r)
DEGREE (L)

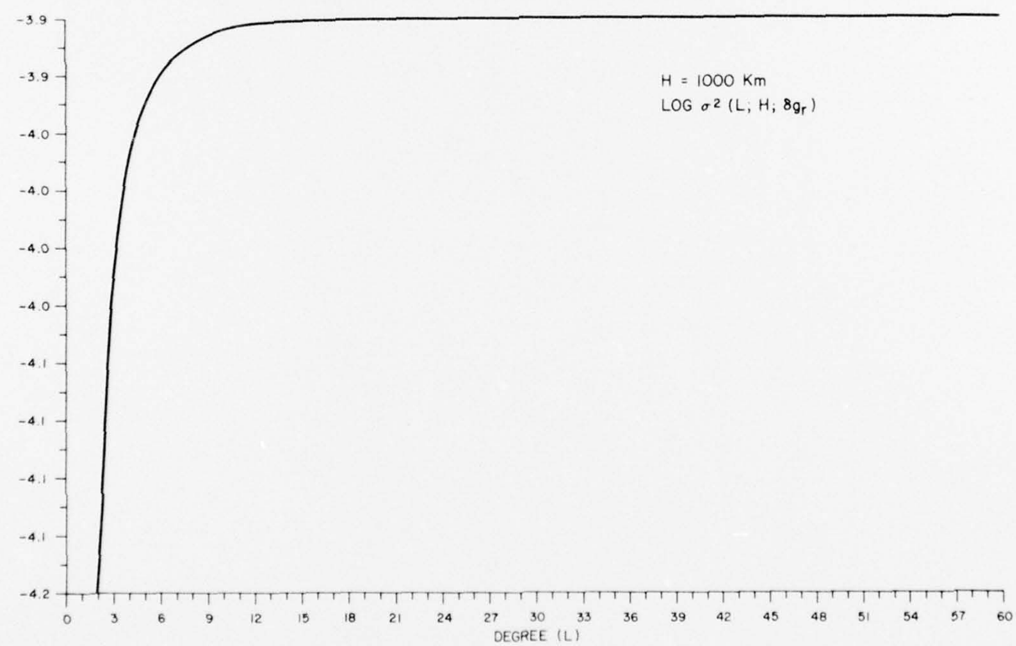

H = 800 Km
LOG σ^2 (L, H, δg_r)
-3.8
-3.8
-3.8
-3.9
-3.9
-3.9
-4.0
-4.0
-4.0
-4.0
-4.1
0
50
100
150
200
250
300
350
400
450
500
DEGREE (L)

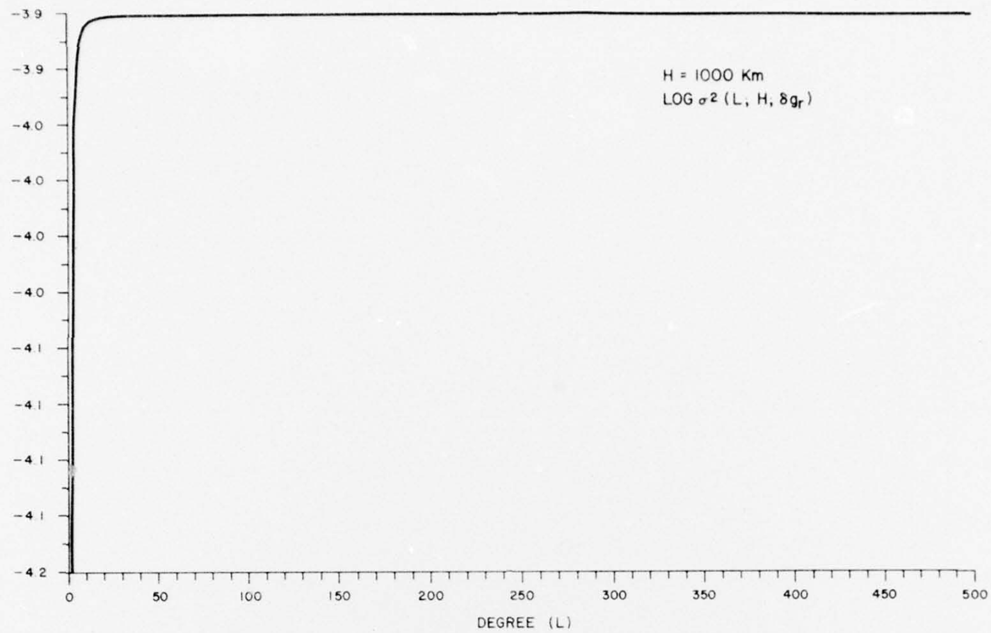

H = 900 Km
LOG $\sigma^2(L, H, \Delta\delta g_r)$
-7.1
-7.3
-7.5
-7.7
-7.8
-8.0
-8.2
-8.3
-8.5
-8.7
-8.9
0 3 6 9 12 15 18 21 24 27 30 33 36 39 42 45 48 51 54 57 60
DEGREE (L)


H = 900 Km
LOG $\sigma^2(L, H, \delta g_r)$
-3.8
-3.9
-3.9
-3.9
-4.0
-4.0
-4.0
-4.0
-4.1
-4.1
-4.1
0 3 6 9 12 15 18 21 24 27 30 33 36 39 42 45 48 51 54 57 60
DEGREE (L)

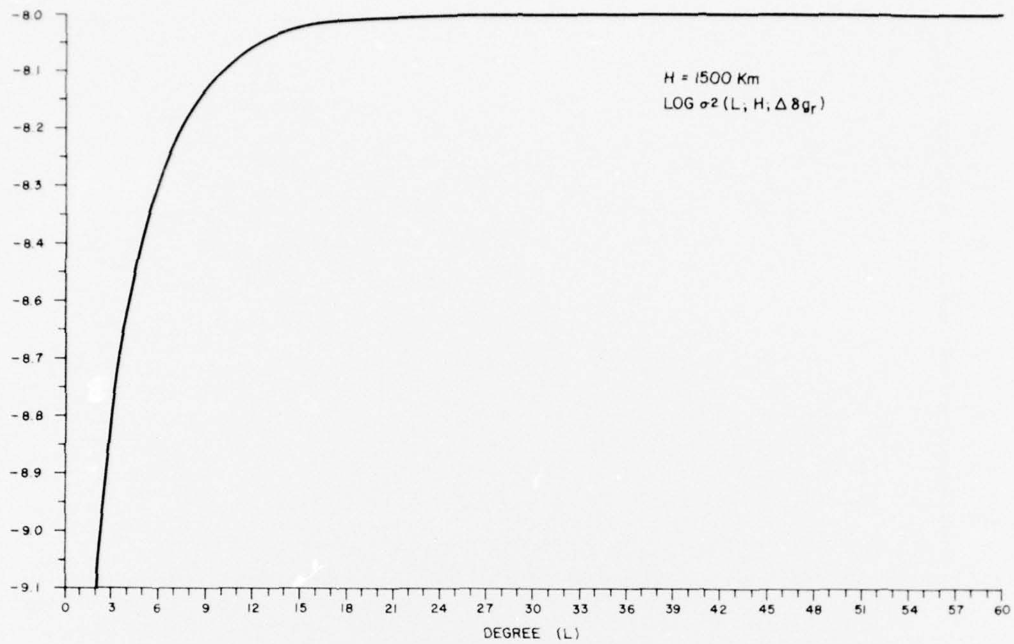

H = 900 Km
LOG $\sigma^2(L, H, \delta g_r)$
-3.8
-3.9
-3.9
-3.9
-4.0
-4.0
-4.0
-4.0
-4.1
-4.1
-4.1
0
50
100
150
200
250
300
350
400
450
500
DEGREE (L)

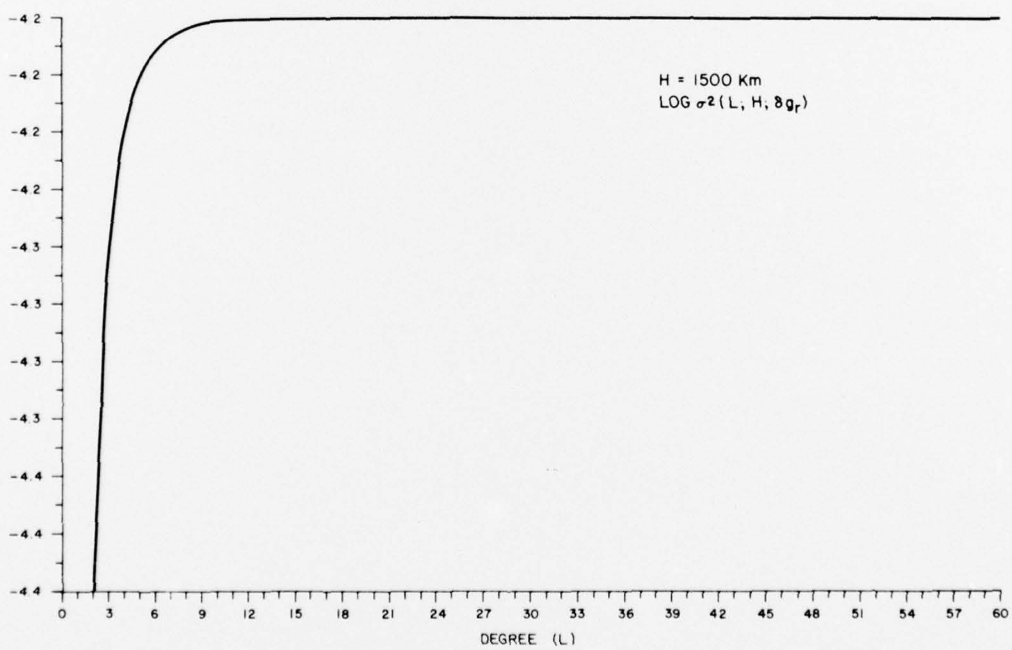

H = 1000 Km
LOG σ^2 (L, H; $\Delta \delta g_r$)
DEGREE (L)

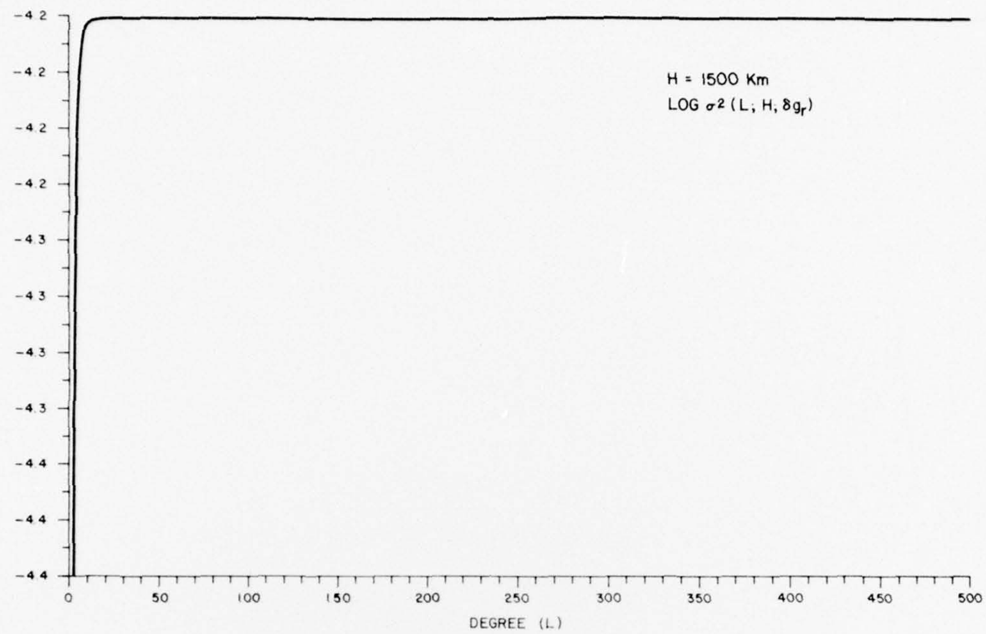

H = 1000 Km
LOG σ^2 (L, H; δg_r)
DEGREE (L)



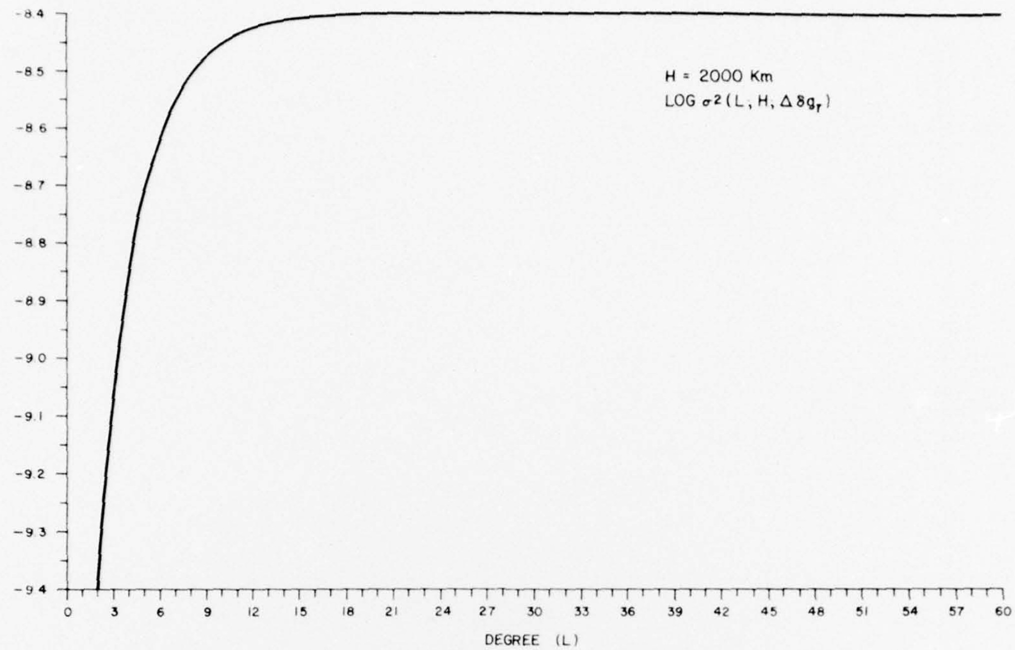
-3.9
-3.9
-4.0
-4.0
-4.0
-4.0
-4.1
-4.1
-4.1
-4.1
-4.2
H = 1000 Km
LOG σ^2 (L, H, δg_r)
0
50
100
150
200
250
300
350
400
450
500
DEGREE (L)

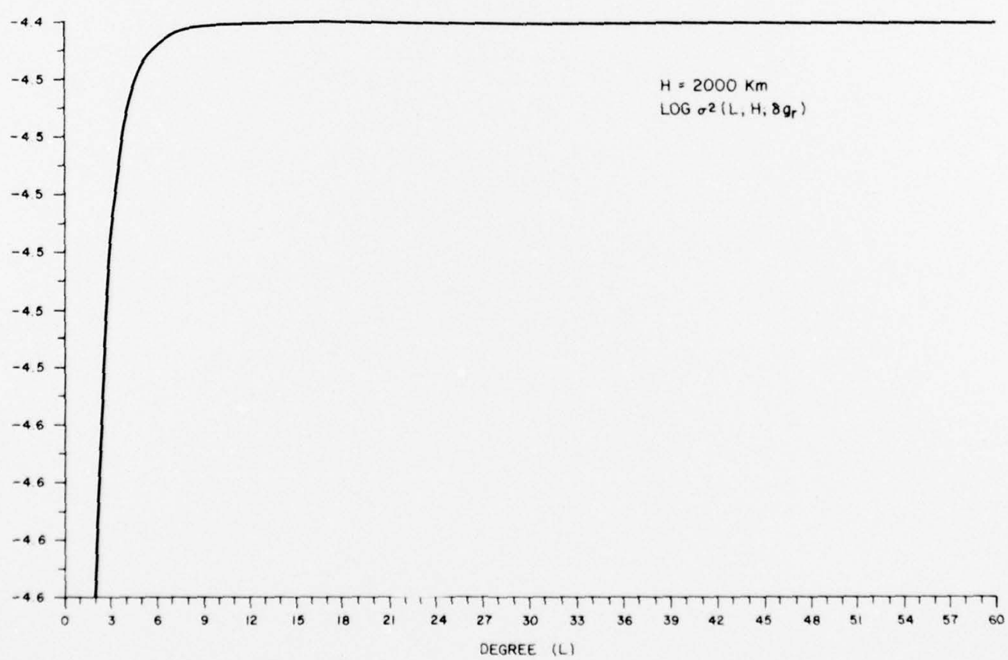

H = 1500 Km
LOG σ2 (L; H; Δ δg_r)
DEGREE (L)

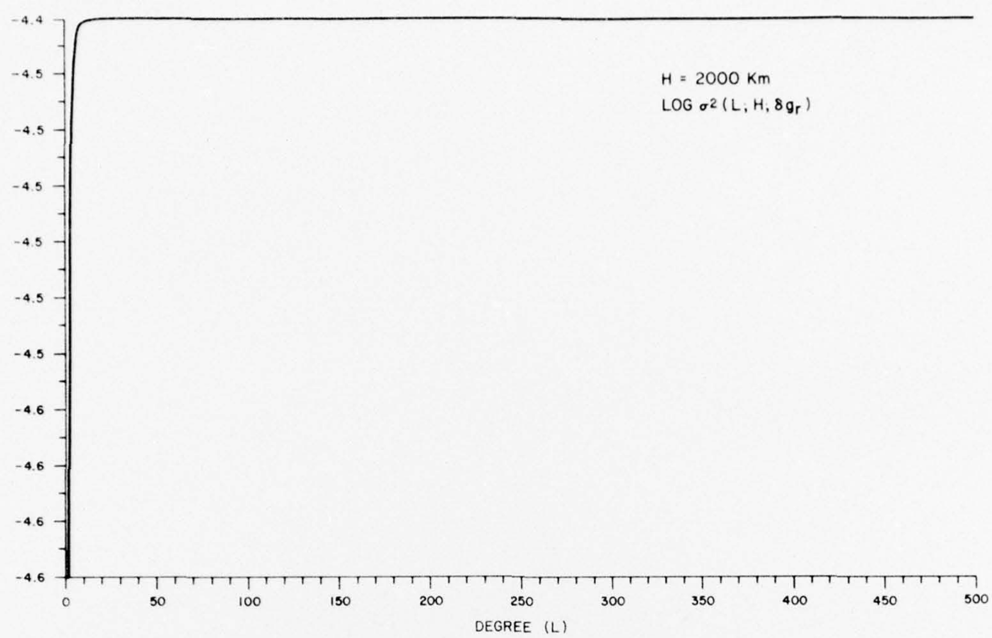

H = 1500 Km
LOG σ2 (L; H; δg_r)
DEGREE (L)



-4.2
-4.2
-4.2
-4.2
-4.3
-4.3
-4.3
-4.3
-4.4
-4.4
-4.4
H = 1500 Km
LOG σ2 (L; H; δgr)
0
50
100
150
200
250
300
350
400
450
500
DEGREE (L)


H = 2000 Km
LOG $\sigma^2(L, H, \Delta\delta g_r)$
DEGREE (L)


H = 2000 Km
LOG $\sigma^2(L, H, \delta g_r)$
DEGREE (L)


H = 2000 Km
LOG $\sigma^2(L; H; \delta g_r)$
-4.4
-4.5
-4.5
-4.5
-4.5
-4.5
-4.5
-4.6
-4.6
-4.6
-4.6
0
50
100
150
200
250
300
350
400
450
500
DEGREE (L)

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